



The 30 Year Horizon

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Volume Bibliography: Axiom Literature Citations

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Grant Keady Ted Kosan Bernhard Kutzler Kaj Laurson Frederic Lehobey Ren-Cang Li Richard Luczak Alasdair McAndrew Edi Meier Victor S. Miller H. Michael Moeller Scott Morrison William Navlor John Nelder Jinzhong Niu Kostas Oikonomou Bill Page Michel Petitot Frederick H. Pitts Claude Quitte Anatoly Raportirenko Guilherme Reis Jean Rivlin Raymond Rogers Philip Santas Gerhard Schneider Frithjof Schulze V. Sima Elena Smirnova Christine Sundaresan Eugene Surowitz James Thatcher Dylan Thurston Themos T. Tsikas Stephen Watt M. Weller Thorsten Werther James T. Wheeler Clifton J. Williamson Robert Wisbauer Knut Wolf Clifford Yapp Richard Zippel

Dan Zwillinger

Wilfrid Kendall Paul Kosinski Tim Lahey George L. Legendre Michel Levaud Rudiger Loos Camm Maguire Bob McElrath Ian Meikle Gerard Milmeister Michael Monagan Joel Moses Patrice Naudin Godfrey Nolan Michael O'Connor Humberto Ortiz-Zuazaga David Parnas Didier Pinchon Jose Alfredo Portes Arthur C. Ralfs Albert D. Rich Huan Ren Nicolas Robidoux Michael Rothstein Alfred Scheerhorn Martin Schoenert Fritz Schwarz Nick Simicich Jonathan Steinbach Robert Sutor Max Tegmark **Balbir Thomas** Steve Toleque Gregory Vanuxem Jaap Weel Mark Wegman Michael Wester John M. Wiley Stephen Wilson Sandra Wityak Yanvang Xiao David Yun Evelyn Zoernack

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New Foreword

On October 1, 2001 Axiom was withdrawn from the market and ended life as a commercial product. On September 3, 2002 Axiom was released under the Modified BSD license, including this document. On August 27, 2003 Axiom was released as free and open source software available for download from the Free Software Foundation's website, Savannah.

Work on Axiom has had the generous support of the Center for Algorithms and Interactive Scientific Computation (CAISS) at City College of New York. Special thanks go to Dr. Gilbert Baumslag for his support of the long term goal.

The online version of this documentation is roughly 1000 pages. In order to make printed versions we've broken it up into three volumes. The first volume is tutorial in nature. The second volume is for programmers. The third volume is reference material. We've also added a fourth volume for developers. All of these changes represent an experiment in print-on-demand delivery of documentation. Time will tell whether the experiment succeeded.

Axiom has been in existence for over thirty years. It is estimated to contain about three hundred man-years of research and has, as of September 3, 2003, 143 people listed in the credits. All of these people have contributed directly or indirectly to making Axiom available. Axiom is being passed to the next generation. I'm looking forward to future milestones.

With that in mind I've introduced the theme of the "30 year horizon". We must invent the tools that support the Computational Mathematician working 30 years from now. How will research be done when every bit of mathematical knowledge is online and instantly available? What happens when we scale Axiom by a factor of 100, giving us 1.1 million domains? How can we integrate theory with code? How will we integrate theorems and proofs of the mathematics with space-time complexity proofs and running code? What visualization tools are needed? How do we support the conceptual structures and semantics of mathematics in effective ways? How do we support results from the sciences? How do we teach the next generation to be effective Computational Mathematicians?

The "30 year horizon" is much nearer than it appears.

Tim Daly CAISS, City College of New York November 10, 2003 ((iHy))

Chapter 1

The Axiom Bibliography

This bibliography covers areas of computational mathematics. Papers which mention Axiom have a "keyword=" entry of "axiomref". Papers we have on site have a "paper=" entry.

The authors are listed in the index. The topic keywords are listed in the index. Algorithms are mentioned in the index.

The **TO** index entry tries to say that the first named algorithm or author has been updated or improved by the second named algorithm or author.

Introduction of special terms (e.g. Toeplitz matrix) may include a paragraph for those unfamiliar with the terms.

Chapter 2

The Bibliography

2.1 Algebra Documentation References

```
— axiom.bib —
@article{Sims71,
  author = "Sims, Charles",
  title = "Determining the Conjugacy Classes of a Permutation Group",
  journal = "Computers in Algebra and Number Theory, SIAM-AMS Proc.",
 volume = "4",
 publisher = "American Math. Soc.",
 year = "1991",
 pages = "191--195",
  comment = "documentation for PermutationGroup"
}
            — axiom.bib —
@article{Worz80,
  author = {W\"orz-Busekros, A.},
 title = "Algebra in Genetics",
 publisher = "Springer-Verlag",
 journal = "Lecture Notes in Biomathematics",
 volume = "36",
 year = "1980",
  comment = "documentation for AlgebraGivenByStructuralConstants"
}
```

```
@article{Reed97,
  author = "Reed, Mary Lynn",
  title = "Algebraic Structure of Genetic Inheritance",
  journal = "Bulletin of the American Mathematical Society",
  year = "1997",
  volume = "34",
 number = "2",
  month = "April",
  pages = "107--130",
  paper = "Reed97.pdf",
  comment = "documentation for AlgebraGivenByStructuralConstants",
  url="http://www.ams.org/bull/1997-34-02/S0273-0979-97-00712-X/S0273-0979-97-00712-X.pdf",
  abstract =
   "In this paper we will explore the nonassociative algebraic structure
    that naturally ocurs as genetic informatin gets passed down through
    the generations. While modern understanding of genetic inheritance
    initiated with the theories of Charles Darwin, it was the Augustinian
    monk Gregor Mendel who began to uncover the mathematical nature of the
    subject. In fact, the symbolism Mendel used to describe his first
    results (e.g. see his 1866 paper {\sl Experiments in
    Plant-Hybridization} is quite algebraically suggestive. Seventy four
    years later, I.M.H. Etherington introduced the formal language of
    abstract algebra to the study of genetics in his series of seminal
    papers. In this paper we will discuss the concepts of genetics that
    suggest the underlying algebraic structure of inheritance, and we will
    give a brief overview of the algebras which arise in genetics and some
    of their basi properties and relationships. With the popularity of
    biologically motivated mathematics continuing to rise, we offer this
    survey article as another example of the breadth of mathematics that
    has biological significance. The most comprehensive reference for the
    mathematical research done in this area (through 1980) is
    W\.orz-Busekros "
}
```

```
@article{Gons71,
  author = "Gonshor, H.",
  title = "Contributions to Genetic Algebras",
```

```
journal = "Proc. Edinburgh Mathmatical Society (Series 2)",
  volume = "17",
  number = "4",
  month = "December",
  year = "1971",
  issn = "1464-3839",
  pages = "289--298",
  doi = "10.1017/S0013091500009548",
  url = "http://journals.cambridge.org/article_S0013091500009548",
  comment = "documentation for AlgebraGivenByStructuralConstants",
  abstract =
    "Etherington introduced certain algebraic methods into the study of
    population genetics. It was noted that algebras arising in genetic
    systems tend to have certain abstract properties and that these can be
    used to give elegant proofs of some classical stability theorems in
    population genetics."
}
```

2.2 Linear Algebra

— axiom.bib —

In the past 10 years there has been substantial activity on the improvement of a solution proposed by Wiedemann in 1986. The main new ingredients are faster pre-conditioners, projections by an entire block of random vectors, Lanczos recurrences, and a connection to

an integer solution to a sparse linear system, are reduced via p-adic

lifting to sparse matrix analysis over a finite field.

}

Kalman realizations of control theory. My talk surveys these developments and describe some major unresolved problems."

— axiom.bib —

@Article{Chen02,

journal = "Linear Algebra and Applications",
year = "2002",

year = "2002",

volume = "343--344",
pages = "119--146",

url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/CEKSTV02.pdf",
paper = "Chen02.pdf",

abstract = "

The main idea of the ''black box'' approach in exact linear algebra is to reduce matrix problems to the computation of minimum polynomials. In most cases preconditioning is necessary to obtain the desired result. Here good preconditioners will be used to ensure geometrical / algebraic properties on matrices, rather than numerical ones, so we do not address a condition number. We offer a review of problems for which (algebraic) preconditioning is used, provide a bestiary of preconditioning problems, and discuss several preconditioner types to solve these problems. We present new conditioners, including conditioners to preserve low displacement rank for Toeplitz-like matrices. We also provide new analyses of preconditioner performance and results on the relations among preconditioning problems and with linear algebra problems. Thus, improvements are offered for the efficiency and applicability of preconditioners. The focus is on linear algebra problems over finite fields, but most results are valid for entries from arbitrary fields."

— axiom.bib —

@InCollection{Kalt11d,

}

author = "Kaltofen, Erich and Storjohann, Arne",

```
title = "The Complexity of Computational Problems in Exact Linear Algebra",
  booktitle = "Encyclopedia of Applied and Computational Mathematics",
  crossref = "EACM",
  year = "2011",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/11/KS11.pdf",
  paper = "Kalt11d.pdf",
  abstract = "
    Computational problems in exact linear algebra including computing an
    exact solution of a system of linear equations with exact scalars,
    which can be exact rational numbers, integers modulo a prime number,
    or algebraic extensions of those represented by their residues modulo
    a minimum polynomial. Classical linear algebra problems are computing
    for a matrix its rank, determinant, characteristic and minimal
    polynomial, and rational canonical form (= Frobenius normal form). For
    matrices with integer and polynomial entries one computes the Hermite
    and Smith normal forms. If a rational matrix is symmetric, one
    determines if the matrix is definite."
}
```

```
@Article{Come12,
  author = "Comer, Matthew T. and Kaltofen, Erich L.",
  title = "On the {Berlekamp}/{Massey} Algorithm and Counting Singular {Hankel}
           Matrices over a Finite Field",
  year = "2012",
  month = "April",
  journal = "Journal of Symbolic Computation",
  volume = "47",
  number = "4",
 pages = "480--491",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/10/CoKa10.pdf",
  paper = "Come12.pdf",
  abstract = "
    We derive an explicit count for the number of singular $n\times n$
    Hankel (Toeplitz) matrices whose entries range over a finite field
    with $q$ elements by observing the execution of the Berlekamp / Massey
    algorithm on its elements. Our method yields explicit counts also when
    some entries above or on the anti-diagonal (diagonal) are fixed. For
    example, the number of singular $n\times n$ Toeplitz matrices with 0's
    on the diagonal is q^{2n-3}+q^{n-1}-q^{n-2}.
```

We also derive the count for all \$n\times n\$ Hankel matrices of rank \$r\$ with generic rank profile, I.e., whose first \$r\$ leading principal submatrices are non-singular and the rest are singular, namely

month = "September",

journal = "ACM Trans. Algorithms",

```
q^r(q-1)^r in the case r < n and q^r(r-1)(q-1)^r in the case
   $r=n$. This result generalizes to block-Hankel matrices as well."
}
           — axiom.bib —
@Article{Kalt13a,
 author = "Kaltofen, Erich and Yuhasz, George",
 title = "A Fraction Free Matrix {Berlekamp}/{Massey} Algorithm",
  journal = "Linear Algebra and Applications",
 year = "2013",
  volume = "439",
  number = "9",
  month = "November",
  pages = "2515--2526",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/08/KaYu08.pdf",
  paper = "Kalt13a.pdf",
  abstract = "
   We describe a fraction free version of the Matrix Berlekamp / Massey
   algorithm. The algorithm computes a minimal matrix generator of
   linearly generated square matrix sequences over an integral
   domain. The algorithm performs all operations in the integral domain,
    so all divisions performed are exact. For scalar sequences, the matrix
   algorithm specializes to a more efficient algorithm than the algorithm
    currently in the literature. The proof of integrality of the matrix
    algorithm gives a new proof of integrality for the scalar
   specialization."
}
           — axiom.bib —
@Article{Kalt13,
  author = "Kaltofen, Erich and Yuhasz, George",
  title = "On The Matrix {Berlekamp}-{Massey} Algorithm",
 year = "2013",
 volume = "9",
 number = "4",
```

```
url = "http://www.math.ncsu.edu/~kaltofen/bibliography/06/KaYu06.pdf",
  paper = "Kalt13.pdf",
  abstract = "
    We analyze the Matrix Berlekamp / Massey algorithm, which generalizes
    the Berlekamp / Massey algorithm [Massey 1969] for computing linear
    generators of scalar sequences. The Matrix Berlekamp / Massey
    algorithm computes a minimal matrix generator of a linearly generated
    matrix sequence and has been first introduced by Rissanen [1972a],
    Dickinson et al. [1974], and Coppersmith [1994]. Our version of the
    algorithm makes no restrictions on the rank and dimensions of the
    matrix sequence. We also give new proofs of correctness and complexity
    for the algorithm, which is based on self-contained loop invariants
    and includes an explicit termination criterion for a given
    determinantal degree bound of the minimal matrix generator"
}
            — axiom.bib —
@InProceedings{Kalt02a,
  author = "Kaltofen, Erich",
  title = "An output-sensitive variant of the baby steps/\allowbreak
           giant steps determinant algorithm",
  booktitle = "Proc. 2002 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC02",
  pages = "138--144",
  year = "2002",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/Ka02.pdf",
 paper = "Kalt02a.pdf",
}
            — axiom.bib —
@InProceedings{Kalt01a,
  author = "Kaltofen, E. and Villard, G.",
  title = "On the complexity of computing determinants",
  booktitle = "Proc. Fifth Asian Symposium on Computer Mathematics
              (ASCM 2001)",
  crossref = "ASCM01",
 pages = "13--27",
  isbn = "981-02-4763-X",
  year = "2001",
```

```
url = "http://www.math.ncsu.edu/~kaltofen/bibliography/01/KaVi01.pdf",
paper = "Kalt01a.pdf",
abstract = "
```

The computation of the determinant of an \$n<table-cell>\times n\$ matrix \$A\$ of numbers or polynomials is a challenge for both numerical and symbolic methods. Numerical methods, such as Clarkson's algorithm [10,7] for the sign of the determinant must deal with conditionedness that determines the number of mantissa bits necessary for obtaining a correct sign. Symbolic algorithms that are based on Chinese remaindering [6,17,Chapter 5.5] must deal with the fact that the length of the determinant in the worse case grows linearly in the dimension of the matrix. Hence the number of modular operations is \$n\$ times the number of arithmetic operations in a given algorithm. Hensel lifting combined with rational number recovery [14,1] has cubic bit complexity in \$n\$, but the algorithm can only determine a factor of the determinant, namely the largest invariant factor. If the matrix is similar to a multiple of the identity matrix, the running time is again that of Chinese remaindering."

```
@Article{Kalt04a,
 author = "Kaltofen, Erich and Villard, Gilles",
 title = "On the Complexity of Computing Determinants",
 journal = "Computational Complexity",
 volume = "13",
 number = "3-4",
 year = "2004",
 pages = "91--130",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/04/KaVi04_2697263.pdf",
 paper = "Kalt04a.pdf",
 abstract = "
   We present new baby steps / giant steps algorithms of asymptotically
    fast running time for dense matrix problems. Our algorithms compute
   the determinant, characteristic polynomial, Frobenius normal form and
    Smith normal form of a dense $n\times n$ matrix $A$ with integer
    entries in (x^{3.2} \log |A|)^{1+o(1)} and
   (x^{2.697263} \log ||A||)^{1+o(1)}
    bit operations; here $||A||$ denotes the largest
    entry in absolute value and the exponent adjustment by "$+o(1)$",
    captures additional factors $C_1 (log n)^{C_2}(loglog ||A||)^{C_3}$
    for positive real constants $C_1$, $C_2$, $C_3$. The bit complexity
    (n^{3.2} \log |A|)^{1+o(1)} results from using the classical cubic
```

}

```
matrix multiplication algorithm. Our algorithms are randomized, and we can certify that the output is the determinant of $A$ in a Las Vegas fashion. The second category of problems deals with the setting where the matrix $A$ has elements from an abstract commutative ring, that is, when no divisions in the domain of entries are possible. We present algorithms that deterministically compute the determinant, characteristic polynomial and adjoint of $A$ with n^{3.2+o(1)} and 0n^{2.697263} ring additions, substractions, and multiplications."
```

— axiom.bib —

```
@InProceedings{Kalt97b,
  author = "Eberly, W. and Kaltofen, E.",
  title = "On Randomized {Lanczos} Algorithms",
  booktitle = "Proc. 1997 Internat. Symp. Symbolic Algebraic Comput.",
  year = "1997",
  crossref = "ISSAC97",
  pages = "176--183",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/97/EbKa97.pdf",
  paper = "Kalt97b.pdf",
  abstract = "
    Las Vegas algorithms that are based on Lanczo's method for solving
    symmetric linear systems are presented and analyzed. These are
    compared to a similar randomized Lanczos algorithm that has been used
    for integer factorization, and to the (provably reliable) algorithm of
    Wiedemann. The analysis suggests that our Lanczos algorithms are
    preferable to several versions of Wiedemann's method for computations
    over large fields, expecially for certain symmetric matrix
    computations."
}
```

The Sylvester matrix is used to compute the **resultant** of two polynomials. The Sylvester matrix is formed from the coefficients of the two polynomials. Given a polynomial with degree m and another of degree n form an $(m+n)\times(m+n)$ matrix by filling the matrix from the upper left corner with the coefficients of the first polynomial then shifting down one row and one column to the right and filling in the coefficients starting there until they hit the right column. Starting at the next row, do the same process for the second polynomial. The determinant of this matrix is the **resultant** of the two polynomials.

For example, given $a_3x^3 + a_2x^2 + a_1x + a_0$ and $b_2x^2 + b_1x + b_0$ the Sylvester matrix is a

 $(3+2) \times (3+2)$ matrix:

$$\begin{bmatrix} a_3 & a_2 & a_1 & a_0 & 0 \\ 0 & a_3 & a_2 & a_1 & a_0 \\ b_2 & b_1 & b_0 & 0 & 0 \\ 0 & b_2 & b_1 & b_0 & 0 \\ 0 & 0 & b_2 & b_1 & b_0 \end{bmatrix}$$

The resultant of these two polynomials (assuming a leading coefficient of 1), is the product of the differences $p_i - q_i$ between the roots of the polynomials. If there are roots in common then the product will contain a 0 and the whole equation reduces to 0. This can be used to determine if two polynomials have common roots.

For example, given a polynomial in x with distinct roots a_1 and a_2 it can be factored as $t1 := (x - a_1)(x - a_2)$.

Given a second polynomial in x with distinct roots b_1 , b_2 , and b_3 it can be factored as $t2 := (x - b_1)(x - b_2)(x - b_3)$.

The Axiom call of resultant(t1, t2, x) is

$$(b_1-a_2)(b_1-a_1)(b_2-a_2)(b_2-a_1)(b_3-a_2)(b_3-a_1)$$

In symbolic form the resultant can show the multiplicity of roots when shown in factored form.

— axiom.bib —

```
@InProceedings{Kalt94c,
  author = "Kaltofen, E.",
  title = "Asymptotically fast solution of {Toeplitz}-like singular
           linear systems",
  booktitle = "Proc. 1994 Internat. Symp. Symbolic Algebraic Comput.",
  pages = "297--304",
  crossref = "ISSAC94",
  vear = "1994",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/94/Ka94_issac.pdf",
  paper = "Kalt94c.pdf",
  abstract = "
    The Toeplitz-likeness of a matrix (Kailath et al. 1979) is the
    generalization of the notion that a matrix is Toeplitz. Block matrices
    with Toeplitz blocks, such as the Sylvester matrix corresponding to
    the resultant of two univariate polynomials, are Toeplitz-like, as are
    products and inverses of Toeplitz-like matrices. The displacement rank
    of a matrix is a measure for the degree of being Toeplitz-like. For
    example, an $r\times s$ block matrix with Toeplitz blocks has
```

displacement rank \$r+s\$ whereas a generic \$N\ximtes N\$ matrix has displacement rank \$N\$. A matrix of displacement rank \$\alpha\$ can be implicitly represented by a sum of \$\alpha\$ matrices, each of which is the product of a lower trainagular and an upper triangular Toeplitz matrices. Such a \$\sigmaLU\$ representation can usually be obtained

```
efficiently."
}
            — axiom.bib —
@Article{Kalt99,
  author = "Kaltofen, E. and Lobo, A.",
  title = "Distributed matrix-free solution of large sparse linear systems over
          finite fields".
  journal = "Algorithmica",
  year = "1999",
 pages = "331--348",
  month = "July--Aug.",
  volume = "24",
  number = "3--4"
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/99/KaLo99.pdf",
 paper = "Kalt99.pdf",
  abstract = "
    We describe a coarse-grain parallel approach for the homogeneous
    solution of linear systems. Our solutions are symbolic, i.e., exact
    rather than numerical approximations. We have performed an outer loop
    parallelization that works well in conjunction with a black box
    abstraction for the coefficient matrix. Our implementation can be run
    on a network cluster of UNIX workstations as well as on an SP-2
    multiprocessor. Task distribution and management are effected through
    MPI and other packages. Fault tolerance, checkpointing, and recovery
    are incorporated. Detailed timings are presented for experiments with
    systems that arise in RSA challenge integer factoring efforts. For
    example, we can solve a 252,222$\times$252,222 system with about 11.04
    million nonzero entries over the Galois field with two elements using
    four processors of an SP-2 multiprocessor, in about 26.5 hours CPU time."
}
            — axiom.bib —
@InProceedings{Kalt96a,
  author = "Kaltofen, E. and Lobo, A.",
  title = "Distributed matrix-free solution of large sparse linear systems
          over finite fields",
```

```
booktitle = "Proc. High Performance Computing '96",
  year = "1996",
  editor = "A. M. Tentner",
  pages = "244--247",
  organization = "Society for Computer Simulation",
  publisher = "Simulation Councils, Inc.",
  address = "San Diego, CA",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/96/KaLo96_hpc.pdf",
  paper = "Kalt96a.pdf",
  abstract = "
    We describe a coarse-grain parallel software system for the
   homogeneous solution of linear systems. Our solutions are symbolic,
    i.e., exact rather than numerical approximations. Our implementation
    can be run on a network cluster of SPARC-20 computers and on an SP-2
    multiprocessor. Detailed timings are presented for experiments with
    systems that arise in RSA challenge integer factoring efforts. For
    example, we can solve a 252,222$\times$252,222 system with about 11.04
    million non-zero entries over the Galois field with 2 elements using 4
   processors of an SP-2 multiprocessor, in about 26.5 hours CPU time."
}
```

```
@InProceedings{Kalt94a,
 author = "Kaltofen, E. and Lobo, A.",
 title = "Factoring high-degree polynomials by the black box
          Berlekamp algorithm",
 booktitle = "Proc. 1994 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC94",
 pages = "90--98",
 year = "1994",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/94/KaLo94.ps.gz",
 paper = "Kalt94a.ps",
 abstract = "
   Modern techniques for solving structured linear systems over finite
    fields, which use the coefficient matrix as a black box and require an
   efficient algorithm for multiplying this matrix by a vector, are
    applicable to the classical algorithm for factoring a univariate
   polynomial over a finite field by Berlekamp (1967 and 1970). We report
    aon a computer implementation of this idea that is based on the
   parallel block Wiedemann linear system solver (Coppersmith 1994 and
   Kaltofen 1993 and 1995). The program uses randomization and we also
    study the expected run time behavior of our method."
```

```
@Article{Kalt95,
  author = "Kaltofen, E.",
  title = "Analysis of {Coppersmith}'s block {Wiedemann} algorithm for the
          parallel solution of sparse linear systems",
  journal = "Math. Comput.",
  year = "1995",
  volume = "64",
 number = "210",
  pages = "777--806",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/Ka95_mathcomp.pdf",
  paper = "Kalt95.pdf",
  abstract = "
    By using projections by a block of vectors in place of a single vector
    it is possible to parallelize the outer loop of iterative methods for
    solving sparse linear systems. We analyze such a scheme proposed by
    Coppersmith for Wiedemann's coordinate recurrence algorithm, which is
    based in part on the Krylov subspace approach. We prove that by use of
    certain randomizations on the input system the parallel speed up is
    roughly by the number of vectors in the blocks when using as many
    processors. Our analysis is valid for fields of entries that have
    sufficiently large cardinality. Our analysis also deals with an
    arising subproblem of solving a singular block Toeplitz system by use
    of the theory of Toeplitz-like matrices."
}
```

```
@Article{Kalt90a,
   author = "Kaltofen, E. and Krishnamoorthy, M.S. and Saunders, B.D.",
   title = "Parallel algorithms for matrix normal forms",
   journal = "Linear Algebra and Applications",
   year = "1990",
   volume = "136",
   pages = "189--208",
   url = "http://www.math.ncsu.edu/~kaltofen/bibliography/90/KKS90.pdf",
   paper = "Kalt90a.pdf",
   abstract = "
    Here we offer a new randomized parallel algorithm that determines the
   Smith normal form of a matrix with entries being univariate
```

}

}

polynomials with coefficients in an arbitrary field. The algorithm has two important advantages over our previous one: the multipliers related the Smith form to the input matrix are computed, and the algorithm is probabilistic of Las Vegas type, i.e., always finds the correct answer. The Smith form algorithm is also a good sequential algorithm. Our algorithm reduces the problem of Smith form computations to two Hermite form computations. Thus the Smith form problem has complexity asymptotically that of the Hermite form problem. We also construct fast parallel algorithms for Jordan normal form and testing similarity of matrices. Both the similarity and non-similarity problems are in the complexity class RNC for the usual coefficient fields, i.e., they can be probabilistically decided in poly-logarithmic time using polynomially many processors."

— axiom.bib —

```
@Article{Kalt87,
```

volume = "8",
pages = "683--690",

url = "http://www.math.ncsu.edu/~kaltofen/bibliography/87/KKS87.pdf",
paper = "Kalt87.pdf",
abstract = "

Boolean circuits of polynomial size and poly-logarithmic depth are given for computing the Hermite and Smith normal forms of polynomial matrices over finite fields and the field of rational numbers. The circuits for the Smith normal form computation are probabilistic ones and also determine very efficient sequential algorithms. Furthermore, we give a polynomial-time deterministic sequential algorithm for the Smith normal form over the rationals. The Smith normal form algorithms are applied to the Rational canonical form of matrices over finite fields and the field of rational numbers."

```
@InProceedings{Kalt92,
  author = "Kaltofen, E. and Pan, V.",
  title = "Processor-efficient parallel solution of linear systems {II}:
           the positive characteristic and singular cases",
  booktitle = "Proc. 33rd Annual Symp. Foundations of Comp. Sci.",
  year = "1992",
  pages = "714--723",
  publisher = "IEEE Computer Society Press",
  address = "Los Alamitos, California",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/92/KaPa92.pdf",
  paper = "Kalt92.pdf",
  abstract = "
    We show that over any field, the solution set to a system of $n$
    linear equations in $n$ unknowns can be computed in parallel with
    randomization simultaneously in poly-logarithmic time in $n$ and with
    only as many processors as are utilized to multiply two $n\times n$
    matrices. A time unit represents an arithmetic operation in the
    field. For singular systems our parallel timings are asymptotically as
    fast as those for non-singular systems, due to our avoidance of binary
    search in the matrix rank problem, except when the field has small
    positive characteristic; in that case, binary search is avoided to a
    somewhat higher processor count measure."
}
```

```
@InProceedings{Kalt91c,
  author = "Kaltofen, E. and Pan, V.",
  title = "Processor efficient parallel solution of linear systems over
           an abstract field",
  booktitle = "Proc. SPAA '91 3rd Ann. ACM Symp. Parallel Algor. Architecture",
  pages = "180--191",
  publisher = "ACM Press",
  year = "1991",
  address = "New York, N.Y.",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/KaPa91.pdf",
  paper = "Kalt91c.pdf",
  abstract = "
   Parallel randomized algorithms are presented that solve
   $n$-dimensional systems of linear equations and compute inverses of
   n\ non-singular matrices over a field in 0((\log n)^2) time,
   where each time unit represents an arithmetic operation in the field
   generated by the matrix entries. The algorithms utilize with a $0(log n)$
   factor as many processors as are needed to multiply two $n\times n$
   matrices. The algorithms avoid zero divisions with controllably
```

high probability provided the \$O(n)\$ random elements used are selected uniformly from a sufficiently large set. For fields of small positive characteristics, the processor count measures of our solutions are somewhat higher."

— axiom.bib —

```
@InProceedings{Kalt91,
  author = "Kaltofen, E. and Saunders, B.D.",
  editor = "H. F. Mattson and T. Mora and T. R. N. Rao",
  title = "On {Wiedemann's} method of solving sparse linear systems",
  booktitle = "Proc. AAECC-9",
  series = "Lect. Notes Comput. Sci.",
  volume = "539",
  pages = "29--38",
  publisher = "Springer-Verlag",
  year = "1991",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/KaSa91.pdf",
  paper = "Kalt91.pdf",
  abstract = "
```

Douglas Wiedemann's (1986) landmark approach to solving sparse linear systems over finite fields provides the symbolic counterpart to non-combinatorial numerical methods for solving sparse linear systems, such as the Lanczos or conjugate gradient method (see Golub and van Loan (1983)). The problem is to solve a sparse linear system, when the individual entries lie in a generic field, and the only operations possible are field arithmethic; the solution is to be exact. Such is the situation, for instance, if one works in a finite field. Wiedemann bases his approach on Krylov subspaces, but projects further to a sequence of individual field elements. By making a link to the Berlekamp / Massey problem from coding theory -- the coordinate recurrences -- and by using randomization an algorithm is obtained with the following property. On input of an \$n\times n\$ coefficient matrix \$A\$ given by a so-called black box, which is a program that can multiply the matrix by a vector (see Figure 1), and of a vector \$b\$, the algorithm finds, with high probability in case the system is solvable, a random solution vector \$x\$ with \$Ax=b\$. It is assumed that the field has sufficiently many elements, say no less than \$50n^2 log(x)\$, otherwise one goes to a finite algebraic extension. The complexity of the method is in the general singular case \$0(n log (n)) calls to the black box for A and an additional $0(n^2$ log(n)^2)\$ field arithmetic operations."

}

```
@article{Wied86,
  author = "Wiedemann, Douglas H.",
  title = "Solving sparse linear equations over finite fields",
  journal = "IEEE Transactions on Information Theory",
  year = "1986",
  volume = "32",
  number = "1",
 pages = "54-62",
  url =
    "http://www.csd.uwo.ca/~moreno/CS424/Ressources/WIEDEMANN-IEE-1986.pdf",
  paper = "Wied86.pdf",
  abstract = "
    A ''coordinate recurrence'' method for solving sparse systems of
    linear equations over finite fields is described. The algorithms
    discussed all require 0(n_1(\omega_n+n_1)\log k n_1) field operations,
    where $n_1$ is the maximum dimension of the coefficient matrix,
    $\omega$ is approximately the number of field operations required to
    apply the matrix to a test vector, and the value of $k$ depends on the
    algorithm. A probabilistic algorithm is shown to exist for finding the
    determinant of a square matrix. Also, probabilistic algorithms are
    shown to exist for finding the minimum polynomial and rank with some
    arbitrarily small possibility of error."
}
```

2.3 Algebraic Algorithms

- axiom.bib -

```
@InCollection{Diaz97,
  author = "Diaz, A. and Kaltofen, E. and Pan, V.",
  title = "Algebraic Algorithms",
  booktitle = "The Computer Science and Engineering Handbook",
  publisher = "CRC Press",
  year = "1997",
  editor = "A. B. Tucker",
  pages = "226--248",
```

```
address = "Boca Raton, Florida",
          chapter = "10",
          keywords = "survey",
          url = "http://www.math.ncsu.edu/~kaltofen/bibliography/97/DKP97.ps.gz",
          paper = "Diaz97.ps",
         ref = "00965",
          abstract = "
                   The title's subject is the algorithmic approach to algebra: arithmetic
                    with numbers, polynomials, matrices, differential polynomials, such as
                    y^{\perp} = 1/2 + x^4/4) y$, truncated series,
                   and algebraic sets, i.e.,
                    quantified expressions such as <table-cell> x \in \mathbb{R}: x^4+\rho \times x
                    which describes a subset of the two-dimensional space with
                    coordinates $p$ and $q$ for which the given quartic equation has a
                    real root. Algorithms that manipulate such objects are the backbone
                    of modern symbolic mathematics software such as the Maple and
                   Mathematica systems, to name but two among many useful systems. This
                    chapter restricts itself to algorithms in four areas: linear matrix
                    algebra, root finding ov univariate polynomials, solution of systems
                    of nonlinear algebraic equations, and polynomial factorization."
}
```

```
@InCollection{Diaz99,
 author = "Diaz, A. and Emiris, I. and Kaltofen, E. and Pan, V.",
 title = "Algebraic Algorithms",
 booktitle = "Algorithms \& Theory of Computation Handbook",
 publisher = "CRC Press",
 year = "1999",
 editor = "M. J. Atallah",
 address = "Boca Raton, Florida",
 pages = "16.1--16.27",
 isbn = "0-8493-2649-4",
 keywords = "survey",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/99/DEKP99.ps.gz",
 paper = "Diaz99.ps",
 abstract = "
   The title's subject is the algorithmic approach to algebra: arithmetic
   with numbers, polynomials, matrices, differential polynomials, such as
    $y^n+(1/2+x^4/4)y$, truncated series, and algebraic sets, i.e.,
    quantified expressions such as $\exists x \in {\bf R}: x^4+p\cdotx+q=0$,
    which describes a subset of the two-dimensional space with
   coordinates $p$ and $q$ for which the given quartic equation has a
    real root. Algorithms that manipulate such objects are the backbone
```

}

}

of modern symbolic mathematics software such as the Maple and Mathematica systems, to name but two among many useful systems. This chapter restricts itself to algorithms in four areas: linear algebra, root finding for univariate polynomials, solution of systems of nonlinear algebraic equations, and polynomial factorization (see section 5 on some pointers to the vast further material on algebraic algorithms and section 2.2 and [Pan 1993] on further applications to the graph and combinatorial computations)."

— axiom.bib —

```
@InCollection{Kalt87a,
   author = "Kaltofen, E.",
   editor = "J. F. Traub",
   title = "Computer algebra algorithms",
   booktitle = "Annual Review in Computer Science",
   pages = "91--118",
   publisher = "Annual Reviews Inc.",
   year = "1987",
   volume = "2",
   address = "Palo Alto, California",
   keywords = "survey,axiomref",
   url = "http://www.math.ncsu.edu/~kaltofen/bibliography/87/Ka87_annrev.pdf",
   paper = "Kalt87a.pdf",
   abstract = "
```

The origins of the discipline of computer algebra can be found in Issac Newton's {\sl Universal Arithmetic} (1728) [130], where methods for methods for manipulating universal mathematical expressions (i.e. formulas containing symbolic indeterminates) and algorithms for solving equations built with these expressions are systematically discussed. One can interpret the misson of computer algebra as the construction of computer systems that enable scientific or engineering users, for instance, to carry out mathematical manipulation automatically. Indeed, systems with this goal already exist, among them {MACSYMA}, {MAPLE}, {muMATH}, {REDUCE}, {SAC/2}, {SCRATCHPAD/II}, and {SMP}. These systems carry out scientific computing tasks, whose results are distinguished from numerical computing in two principle aspects."

2.4 Sparse Linear Systems

— axiom.bib —

```
@InProceedings{Kalt96b,
  author = "Kaltofen, E.",
  title = "Blocked iterative sparse linear system solvers for finite fields",
  booktitle = "Proc. Symp. Parallel Comput. Solving Large Scale Irregular
               Applic. (Stratagem '96)",
  editor = "C. Roucairol",
  publisher = "INRIA",
  address = "Sophia Antipolis, France",
  pages = "91--95",
  year = "1996",
  keywords = "survey",
  url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/96/Ka96_stratagem.ps.gz",
  paper = "Kalt96b.ps",
  abstract = "
    The problem of solving a large sparse or structured system of linear
    equations in the symbolic context, for instance when the coefficients
    lie in a finite field, has arisen in several applications. A famous
    example are the linear systems of {\bf F}_2, the field with 2
    elements, that arise in sieve based integer factoring algorithms. For
    example, for the factorization of the RSA-130 challenge number several
    column dependencies of a $3504823\times 3516502$ matrix with an
    average of $39.4.$ non-zero entries per column needed to be computed
    \[10] . A second example is the Berlekamp polynomial factorization
    algorithm [6]. In that example, the matrix is not explicitly
    constructed, but instead a fast algorithm for performing the matrix
    times vector product is used. Further examples for such ''black box
    matrices' arise in the power series solutoin of algebraic or
    differential equations by undetermined coefficients. The arising
    linear systems for the coefficients usually have a distinct structure
    that allows a fast coefficient matrix times vector product."
```

2.5 Matrix Determinants

— axiom.bib —

@Article{Kalt04,

}

```
author = "Kaltofen, E. and Villard, G.",
  title = "Computing the sign or the value of the determinant of an integer
           matrix, a complexity survey",
  journal = "J. Computational Applied Math.",
  volume = "162",
  number = "1",
  month = "January",
 pages = "133--146",
  year = "2004",
  keywords = "survey",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/KaVi02.pdf",
  paper = "Kalt04.pdf",
  abstract = "
    Computation of the sign of the determinant of a matrix and the
    determiant itself is a challenge for both numerical and exact
    methods. We survey the complexity of existing methods to solve these
    problems when the input is an $n\times n$ matrix $A$ with integer
    entries. We study the bit-complexities of the algorithms
    asymptotically in $n$ and the norm $A$. Existing approaches rely on
    numerical approximate computations, on exact computations, or on both
    types of arithmetic in combination."
}
```

2.6 Open Problems

```
- axiom.bib -
```

```
@Article{Kalt00,
  author = "Kaltofen, E.",
  title = "Challenges of Symbolic Computation My Favorite Open Problems",
  journal = "Journal of Symbolic Computation",
  volume = "29",
 number = "6",
  pages = "891--919",
  year = "2000",
  keywords = "survey",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/2K/Ka2K.pdf",
  paper = "Kalt00.pdf",
  abstract = "
    The success of the symbolic mathematical computation discipline is
    striking. The theoretical advances have been continuous and significant:
    Gr{\"o}bner bases, the Risch integration algorithm, integer lattice
    basis reduction, hypergeometric summation algorithms, etc. From the
    beginning in the early 60s, it has been the tradition of our discipline
```

}

to create software that makes our ideas readily available to a scientists, engineers, and education: {SAC-1}, {Reduce}, {Macsyma}, etc. The commercial viability of our system products is proven by Maple and Mathematica.

Today's user communities of symbolic computation systems are diverse: educators, engineers, stock market analysts, etc. The mathematics and computer science in the design and implementation of our algorithms are sophisticated. The research challenges in symbolic computation at the close of the 20th century are formidable.

I state my favorite eight open problems in symbolic computations. They range from problems in symbolic /numeric computing, symbolic algorithm synthesis, to system component construction. I have worked on seven of my problems and borrowed one from George Collins. I present background to each of my problems and a clear-cut test that evaluates whether a proposed attack has solved one of my problems. An additional ninth open problem by Rob Corless and David Jeffrey on complex function semantics is given in an appendix."

2.7 Parallel Evaluation

```
@InCollection{Kalt93a,
 author = "Kaltofen, E.",
 editor = "J. Reif",
 title = "Dynamic parallel evaluation of computation {DAG}s",
 booktitle = "Synthesis of Parallel Algorithms",
 pages = "723--758",
 publisher = "Morgan Kaufmann Publ.",
 year = "1993",
 address = "San Mateo, California",
 keywords = "survey",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/93/Ka93_synthesis.ps.gz",
 paper = "Kalt93a.ps",
 abstract = "
   One generic parallel evaluation scheme for algebraic objects, that of
    evaluating algebraic computation trees or formulas, is presented by
   Miller in a preceding chapter of this book. However, there are basic
    algebraic functions for which the tree model of computation seems not
    sufficient to allow an eficient -- even sequential -- decision-free
```

}

algebraic computation. The formula model essentially restricts the use of an intermediate result to a single place, because the parse tree nodes have fan-out one. If an intermediate result participates in the computations of several further nodes, in the tree model it must be recomputed anew for each of these nodes. It is a small formal change to allow node values to propagate to more than one node deeper level of the computation. Thus we obtain the {\sl algebraic circuit model}, which is equivalent to the {\sl straight-line program model}."

2.8 Hybrid Symbolic/Numeric

— axiom.bib —

```
@InProceedings{Kalt06,
   author = "Kaltofen, Erich and Zhi, Lihong",
   title = "Hybrid Symbolic-Numeric Computation",
   year = "2006",
   booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'06",
   crossref = "ISSAC06",
   pages = "7",
   url = "http://www.math.ncsu.edu/~kaltofen/bibliography/06/KaZhi06.pdf",
   paper = "Kalt06.pdf",
   abstract = "
```

Several standard problems in symbolic computation, such as greatest common divisors and factorization of polynomials, sparse interpolation, or computing solutions to overdetermined systems of polynomial equations have non-trivial solutions only if the input coefficients satisfy certain algebraic constraints. Errors in the coefficients due to floating point round-off or through physical measurement thus render the exact symbolic algorithms unusable. By symbolic-numeric methods one computes minimal deformations of the coefficients that yield non-trivial results. We will present hybrid algorithms and benchmark computations based on Gauss-Newton optimazation, singular value decomposition (SVD) and structure-preserving total least squares (STLS) fitting for several of the above problems.

A significant body of results to solve those ''approximate computer algebra'' problems has been discovered in the past 10 years. In the Computer Algebra Handbook the section on ''Hybrid Methods'' concludes as follows [2]: ''The challenge of hybrid symbolic-numeric algorithms is to explore the effects of imprecision, discontinuity, and algorithmic complexity by applying mathematical optimization,

7

perturbation theory, and inexact arithmetic and other tools in order to solve mathematical problems that today are not solvable by numeriical or symbolic methods alone." The focus of our tutorial is on how to formulate several approximate symbolic computation problems as numerical problems in linear algebra and optimization and on software that realizes their solutions."

— axiom.bib —

@InProceedings{Hutt10,

We give a stability criterion for real polynomial inequalities with floating point or inexact scalars by estimating from below or computing the radius of semdefiniteness. That radious is the maximum deformation of the polynomial coefficent vector measured in a weighted Euclidean vector norm within which the inequality remains true. A large radius means that the inequalities may be considered numerically valid.

The radius of positive (or negative) semidefiniteness is the distance to the nearest polynomial with a real root, which has been thoroughly studied before. We solve this problem by parameterized Lagrangian multipliers and Karush-Kuhn-Tucker conditions. Our algorithms can compute the radius for several simultaneous inequalities including possibly additional linear coefficient constraints. Our distance measure is the weighted Euclidena coefficient norm, but we also discuss several formulas for the weighted infinity and 1-norms.

The computation of the nearest polynomial with a real root can be interpreted as a dual of Seidenberg's method that decides if a real hypersurface contains a real point. Sums-of-squares rational lower bound certificates for the radius of semidefinitesness provide a new approach to solving Seidenberg's problem, especially when the coeffcients are numeric. They also offer a surprising alternative sum-of-squares proof for those polynomials that themselves cannot be

pages = "1--15",

```
represented by a polynomial sum-of-squares but that have a positive
    distance to the nearest indefinte polynomial."
}
            — axiom.bib —
@InProceedings{Kalt09,
  author = "Kaltofen, Erich and Yang, Zhengfeng and Zhi, Lihong",
  title = "A Proof of the {Monotone Column Permanent (MCP) Conjecture} for
           Dimension 4 via Sums-Of-Squares of Rational Functions",
  year = "2009",
  booktitle = "Proc. 2009 Internat. Workshop on Symbolic-Numeric Comput.",
  crossref = "SNCO9",
  pages = "65--69",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/09/KYZ09.pdf",
  paper = "Kalt09.pdf",
  abstract = "
    For a proof of the monotone column permanent (MCP) conjecture for
    dimension 4 it is sufficient to show that 4 polynomials, which come
    rom the permanents of real marices, are nonnegative for all real
    values of the variables, where the degrees and the number of the
    variables of these polynomials are all 8. Here we apply a hybrid
    symbolic-numerical algorithm for certifying that these polynomials can
    be written as an exact fraction of two polynomial sums-of-squares
    (SOS) with rational coefficients."
}
            — axiom.bib —
@Article{Kalt12,
  author = "Kaltofen, Erich L. and Li, Bin and Yang, Zhengfeng and
           Zhi, Lihong",
  title = "Exact Certification in Global Polynomial Optimization
          Via Sums-Of-Squares of Rational Functions
          with Rational Coefficients",
  year = "2012",
  month = "January",
  journal = "Journal of Symbolic Computation",
  volume = "47",
  number = "1",
```

url = "http://www.math.ncsu.edu/~kaltofen/bibliography/09/KLYZ09.pdf",

}

paper = "Kalt12.pdf",
abstract = "

We present a hybrid symbolic-numeric algorithm for certifying a polynomial or rational function with rational coefficients to be non-negative for all real values of the variables by computing a representation for it as a fraction of two polynomial sum-of-squares (SOS) with rational coefficients. Our new approach turns the earlier methods by Peyrl and Parrilo and SCN'07 and ours at ISSAC'08 both based on polynomial SOS, which do not always exist, into a universal algorithm for all inputs via Artin's theorem.

Furthermore, we scrutinize the all-important process of converting the numerical SOS numerators and denomiators produced by block semidefinite programming into an exact rational identity. We improve on our own Newton iteration-based high precision refinement algorithm by compressing the initial Gram matrices and by deploying rational vector recovery aside from orthogonal projection. We successfully demenstrate our algorithm on 1. various exceptional SOS problems with necessary polynomial denominators from the literature and on 2. very large (thousands of variables) SOS lower bound certificates for Rump's model problem (up to \$n=18\$, factor degree \$=17\$)."

— axiom.bib —

@InProceedings{Kalt08b,

year = "2008",

booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'08", crossref = "ISSAC08",

pages = "155--163",

url = "http://www.math.ncsu.edu/~kaltofen/bibliography/08/KLYZ08.pdf",
paper = "Kalt08b.pdf",

abstract = "

We generalize the technique by Peyrl and Parillo [Proc. SNC 2007] to computing lower bound certificates for several well-known factorization problems in hybrid symbolic-numeric computation. The idea is to transform a numerical sum-of-squares (SOS) representation of a positive polynomial into an exact rational identity. Our algorithms successfully certify accurate rational lower bounds near the irrational global optima for benchmark approximate polynomial greatest common divisors and multivariate polynomial irreducibility radii from the literature, and factor coefficient bounds in the

setting of a model problem by Rump (up to \$n=14\$, factor degree \$=13\$).

The numeric SOSes produced by the current fixed precision semi-definite programming (SDP) packages (SeDuMi, SOSTOOLS, YALMIP) are usually too coarse to allow successful projection to exact SOSes via Maple 11's exact linear algebra. Therefore, before projection we refine the SOSes by rank-preserving Newton iteration. For smaller problems the starting SOSes for Newton can be guessed without SDP (''SDP-free SOS''), but for larger inputs we additionally appeal to sparsity techniques in our SDP formulation."

— axiom.bib —

@InProceedings{Kalt06b,

}

We consider the problem of computing minimal real or complex deformations to the coefficients in a list of relatively prime real or complex multivariate polynomials such that the deformed polynomials have a greatest common divisor (GCD) of a least a given degree \$k\$. In addition, we restrict the deformed coefficients by a given set of linear constraints, thus introducing the {\sl linearly constrained aproximate GCD} problem. We present an algorithm based on a version of the structured total least norm (STLN) method and demonstrate, on a diverse set of benchmark polynomials, that the algorithm in practice computes globally minimal approximations. As an application of the linearly constrained approximate GCD problem, we present an STLN-based method that computes for a real or complex polynomial the nearest real or complex polynomial the nearest real or complex polynomial that has a root of multiplicity at least \$k\$. We demonstrate that the algorithm in practice computes, on the benchmark polynomials given in the literate, the known globally optimal nearest singular polynomials. Our algorithms can handle, via randomized preconditioning, the difficult case when the nearest solution to a list of real input polynomials actually has non-real complex coefficients."

}

— axiom.bib —

```
@InCollection{Kalt05,
  author = "Kaltofen, Erich and Yang, Zhengfeng and Zhi, Lihong",
  title = "Structured Low Rank Approximation of a {Sylvester} Matrix",
  booktitle = "Symbolic-Numeric Computation",
  crossref = "SNCO6",
  pages = "69--83",
  year = "2005",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/05/KYZ05.pdf",
  paper = "Kalt05.pdf",
  abstract = "
   The task of determining the approximate greatest common divisor (GCD)
    of univariate polynomials with inexact coefficients can be formulated
    as computing for a given Sylvester matrix a new Sylvester matrix of
    lower rank whose entries are near the corresponding entries of that
    input matrix. We solve the approximate GCD problem by a new method
    based on structured total least norm (STLN) algorithms, in our case
    for matrices with Sylvester structure. We present iterative algorithms
    that compute an approximate GCD and that can certify an approximate
    $\epsilon$-GCD when a tolerence $\epsilon$ is given on input. Each
    single iteration is carried out with a number of floating point
    operations that is of cubic order in the input degrees. We also
    demonstrate the practical performance of our algorithms on a diverse
    set of univariate pairs of polynomials."
}
```

— axiom.bib —

```
@InProceedings{Kalt03a,
   author = "Kaltofen, Erich and May, John",
   title = "On Approximate Irreducibility of Polynomials in Several Variables",
   year = "2003",
   booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'03",
   crossref = "ISSAC03",
   pages = "161--168",
   url = "http://www.math.ncsu.edu/~kaltofen/bibliography/03/KM03.pdf",
   paper = "Kalt03a.pdf",
   abstract = "
   We study the problem of bounding all factorizable polynomials away
   from a polynomial that is absolutely irreducible. Such separation
```

bounds are useful for testing whether a numerical polynomial is absolutely irreducible, given a certain tolerance on its coefficients Using an absolute irreducibility criterion due to Ruppert, we are able to find useful separation bounds, in several norms, for bivariate polynomials. We also use Ruppert's criterion to derive new, more effective Noether forms for polynomials of arbitrarily many variables. These forms lead to small separation bounds for polynomials of arbitrarily many variables."

— axiom.bib —

@InProceedings{Gao04a,

}

author = "Gao, Shuhong and Kaltofen, Erich and May, John P. and Yang, Zhengfeng and Zhi, Lihong",

year = "2004",

booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'04",
crossref = "ISSAC04",

pages = "167--174",

url = "http://www.math.ncsu.edu/~kaltofen/bibliography/04/GKMYZ04.pdf",
paper = "Gao04a.pdf",

abstract = "

}

The input to our algorithm is a multivariate polynomial, whose complex rational coefficient are considered imprecise with an unknown error that causes \$f\$ to be irreducible over the complex numbers {\bf C}. We seek to perturb the coefficients by a small quantity such that the resulting polynomial factors over {\bf C}. Ideally, one would like to minimize the perturbation in some selected distance measure, but no efficient algorithm for that is known. We give a numerical multivariate greatest common divisor algorithm and use it on a numerical variant of algorithms by W. M. Ruppert and S. Gao. Our numerical factorizer makes repeated use of singular value decompositions. We demonstrate on a significant body of experimental data that our algorithm is practical and can find factorizable polynomials within a distance that is about the same in relative magnitude as the input error, even when the relative error in the input is substantial (\$10^{-3})."

— axiom.bib —

```
@Article{Kalt08,
  author = "Kaltofen, Erich and May, John and Yang, Zhengfeng and Zhi, Lihong",
  title = "Approximate Factorization of Multivariate Polynomials Using
           Singular Value Decomposition",
  year = "2008",
  journal = "Journal of Symbolic Computation",
  volume = "43",
  number = 5,
  pages = "359--376",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/07/KMYZ07.pdf",
 paper = "Kalt08.pdf",
            — axiom.bib —
@InProceedings{Hitz99,
  author = "Hitz, M.A. and Kaltofen, E. and Lakshman, Y.N.",
  title = "Efficient Algorithms for Computing the Nearest Polynomial
           With A Real Root and Related Problems",
  booktitle = "Proc. 1999 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC99",
  pages = "205--212",
  year = "1999",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/99/HKL99.pdf",
  paper = "Hitz99.pdf",
            — axiom.bib —
@InProceedings{Hitz98,
  author = "Hitz, M. A. and Kaltofen, E.",
  title = "Efficient Algorithms for Computing the Nearest Polynomial
           with Constrained Roots",
  booktitle = "Proc. 1998 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC98",
  year = "1998",
  pages = "236--243",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/98/HiKa98.pdf",
  paper = "Hitz98.pdf",
}
```

2.9 Software Systems

```
— axiom.bib —
@InProceedings{Diaz91,
  author = "Diaz, A. and Kaltofen, E. and Schmitz, K. and Valente, T.",
  title = "DSC A System for Distributed Symbolic Computation",
  booktitle = "Proc. 1991 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC91",
  pages = "323--332",
 year = "1991",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/DKSV91.pdf",
 paper = "Diaz91.pdf",
            — axiom.bib —
@InProceedings{Chan94,
  author = "Chan, K.C. and Diaz, A. and Kaltofen, E.",
  editor = "R. J. Lopez",
  title = "A distributed approach to problem solving in Maple",
  booktitle = "Maple V: Mathematics and its Application",
 pages = "13--21",
 publisher = {Birkh\"auser},
 year = "1994",
  series = "Proceedings of the Maple Summer Workshop and Symposium (MSWS'94)",
  address = "Boston",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/94/CDK94.ps.gz",
 paper = "Chan94.ps",
            — axiom.bib —
@InProceedings{Duma02,
  author = "Dumas, J.-G. and Gautier, T. and Giesbrecht, M. and Giorgi, P.
            and Hovinen, B. and Kaltofen, E. and Saunders, B.D. and
            Turner, W.J. and Villard, G.",
  title = "{LinBox}: A Generic Library for Exact Linear Algebra",
  booktitle =
                 "Proc. First Internat. Congress Math. Software ICMS 2002,
                  Beijing, China",
```

```
"ICMS02",
  crossref =
 pages =
                  "40--50",
 year = "2002",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/Detal02.pdf",
 paper = "Duma02.pdf",
           — axiom.bib —
@InProceedings{Kalt05a,
 author = "Kaltofen, Erich and Morozov, Dmitriy and Yuhasz, George",
 title = "Generic Matrix Multiplication and Memory Management in {LinBox}",
 year = "2005",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'05",
 crossref = "ISSACO5",
 pages = "216--223",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/05/KMY05.pdf",
 paper = "Kalt05a.pdf",
           — axiom.bib —
@InProceedings{Diaz98,
  author = "Diaz, A. and Kaltofen, E.",
  title = "{FoxBox}, a System for Manipulating Symbolic Objects in Black Box
          Representation",
 booktitle = "Proc. 1998 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC98",
 year = "1998",
 pages = "30--37",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/98/DiKa98.pdf",
 paper = "Diaz98.pdf",
           — axiom.bib —
@InProceedings{Diaz93,
```

```
author = "Diaz, A. and Kaltofen, E. and Lobo, A. and Valente, T.",
  editor = "A. Miola",
  title = "Process scheduling in {DSC} and the large sparse linear
          systems challenge",
  booktitle = "Proc. DISCO '93",
  series = "Lect. Notes Comput. Sci.",
  pages = "66--80",
 year = "1993",
  volume = "722",
 publisher = "Springer-Verlag",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/93/DHKLV93.pdf",
 paper = "Diaz93.pdf",
            — axiom.bib —
@Article{Diaz95a,
  author = "Diaz, A. and Hitz, M. and Kaltofen, E. and Lobo, A. and
           Valente, T.",
  title = "Process scheduling in {DSC} and the large sparse linear
          systems challenge",
  journal = "Journal of Symbolic Computing",
  year = "1995",
  volume = "19",
 number = "1--3",
 pages = "269--282",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/DHKLV95.pdf",
 paper = "Diaz95a.pdf",
}
            — axiom.bib —
@Article{Free88,
  author = "Freeman, T.S. and Imirzian, G. and Kaltofen, E. and
           Yagati, Lakshman",
  title = "DAGWOOD: A system for manipulating polynomials given by
          straight-line programs",
  journal = "ACM Trans. Math. Software",
 year = "1988",
 volume = "14",
 number = "3",
 pages = "218--240",
```

```
url = "http://www.math.ncsu.edu/~kaltofen/bibliography/88/FIKY88.pdf",
  paper = "Free88.pdf",
}
```

2.10 The Seven Dwarfs

2.11 Solving Systems of Equations

```
faster and more correct, than MACSYMA's solve." \mbox{\}}
```

2.12 Numerical Algorithms

```
— ignore —
{Bro99,
  author = "Bronstein, Manuel",
  title = "Fast Deterministic Computation of Determinants of Dense Matrices",
  url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
  paper = "Bro99.pdf",
  abstract = "
    In this paper we consider deterministic computation of the exact
    determinant of a dense matrix $M$ of integers. We present a new
    algorithm with worst case complexity
    [0(n^4(\log n + \log \vee erb?||M||?) + x^3 \log^2 \vee erb?||M||?)],
    where $n$ is the dimension of the matrix
    and \verb?||M||? is a bound on the entries in $M$, but with
    average expected complexity
    [0(n^4+m^3(\log n + \log \vee P)/|M||?)^2)],
    assuming some plausible properties about the distribution of $M$.
    We will also describe a practical version of the algorithm and include
    timing data to compare this algorithm with existing ones. Our result
    does not depend on ''fast'' integer or matrix techniques."
}
            — axiom.bib —
@misc{Fate13,
  author = "Fateman, Richard J.",
  title = "Interval Arithmetic, Extended Numbers and Computer Algebra Systems",
  year = "2013",
  paper = "Fate13.pdf",
  url = "http://www.cs.berkeley.edu/~fateman/papers/interval.pdf",
  keywords = "axiomref",
  abstract =
    "Many ambitious computer algebra systems were initially designed in a
    flush of enthusiasm, with the goal of automating any symbolic
    mathematical manipulation ''correctly''. Historically, this approach
```

results in programs that implicitly used certain identities to simplify expressions. These identities, which very likely seemed universally true to the programmers in the heat of writing the CAS (and often were true in well-known abstract algebraic domains) later neede re-examination when such systems were extended for dealing with kinds of objects unanticipated in the original design. These new objects are generally introduced to the CAS by extending "'generically' the arithmetic of other operations. For example, approximate floats do not have the mathematical properties of exact integers or rationals. Complex numbers may strain a system designed for real-valued variables. In the situation examined here, we consider two categories of ''extended'' numbers: \$\infty\$ and {\sl undefined}, and real intervals. We comment on issues raised by these two troublesome notions, how their introduction into a computer algebra system may require a (sometimes painful) reconsideration and redesign of parts of the program, and how they are related. An alternative (followed most notably by the Axiom system is to essentially envision a "meta" CAS defined in terms of categories and inheritance with only the most fundamental built-in concepts; from these one can build many variants of specific CAS features. This approach is appealing but can fail to accommodate extensions that violate some mathematical tenets in the cause of practicality."

— ignore —

{Kel00,

}

author = "Kelsey, Tom",

title = "Exact Numerical Computation via Symbolic Computation",
url = "http://tom.host.cs.st-andrews.ac.uk/pub/ccapaper.pdf",

paper = "Kel00.pdf",

abstract = "

We provide a method for converting any symbolic algebraic expression that can be converted into a floating point number into an exact numeric representation. We use this method to demonstrate a suite of procedures for the representation of, and arithmetic over, exact real numbers in the Maple computer algebra system. Exact reals are represented by potentially infinite lists of binary digits, and interpreted as sums of negative powers of the golden ratio."

}

```
— axiom.bib —
```

```
@article{Stou07,
  author = "Stoutemyer, David R.",
  title = "Useful Computations Need Useful Numbers",
  year = "2007",
 publisher = "ACM",
  journal = "Communications in Computer Algebra",
  volume = "41",
  number = "3",
  abstract =
    "Most of us have taken the exact rational and approximate numbers in
    our computer algebra systems for granted for a long time, not thinking
    to ask if they could be significantly better. With exact rational
    arithmetic and adjustable-precision floating-point arithmetic to
    precision limited only by the total computer memory or our patience,
    what more could we want for such numbers? It turns out that there is
    much that can be done that permits us to obtain exact results more
    often, more intelligible results, approximate results guaranteed to
    have requested error bounds, and recovery of exact results from
    approximate ones."
}
            — ignore —
  author = "Yang, Xiang and Mittal, Rajat",
  title = "Acceleration of the Jacobi iterative method by factors exceeding 100
           using scheduled relation",
"http://engineering.jhu.edu/fsag/wp-content/uploads/sites/23/2013/10/JCP_revised_WebPost.pdf",
 paper = "Yang14.pdf",
```

2.13 Special Functions

- ignore -

{Corl0,

```
author = "Corless, Robert M. and Jeffrey, David J. and Watt, Stephen M.
            and Bradford, Russell and Davenport, James H.",
  title = "Reasoning about the elementary functions of complex analysis",
  url = "http://www.csd.uwo.ca/~watt/pub/reprints/2002-amai-reasoning.pdf",
  paper = "Corl05.pdf",
  abstract = "
    There are many problems with the simplification of elementary
    functions, particularly over the complex plane. Systems tend to make
    "howlers" or not to simplify enough. In this paper we outline the
    "unwinding number" approach to such problems, and show how it can be
    used to prevent errors and to systematise such simplification, even
    though we have not yet reduced the simplification process to a
    complete algorithm. The unsolved problems are probably more amenable
    to the techniques of artificial intelligence and theorem proving than
    the original problem of complex-variable analysis."
}
            — ignore —
{Ng68,
  author = "Ng, Edward W. and Geller, Murray",
  title = "A Table of Integrals of the Error functions",
  url = "http://nvlpubs.nist.gov/nistpubs/jres/73B/jresv73Bn1p1_A1b.pdf",
  paper = "Ng68.pdf",
  abstract = "
   This is a compendium of indefinite and definite integrals of products
    of the Error functions with elementary and transcendental functions."
```

2.14 Exponential Integral $E_1(x)$

```
-- ignore --

{Gel169,
    author = "Geller, Murray and Ng, Edward W.",
    title = "A Table of Integrals of the Exponential Integral",
    url = "http://nvlpubs.nist.gov/nistpubs/jres/73B/jresv73Bn3p191_A1b.pdf",
    paper = "Gel169.pdf",
    abstract = "

This is a compendium of indefinite and definite integrals of products
```

```
of the Exponential Integral with elementary or transcendental functions." \}
```

— axiom.bib —

A four-parameter fit is developed for the class of integrals known as the exponential integral (real branch). Unlike other fits that are piecewise in nature, the current fit to the exponential integral is valid over the complete domain of the function (compact) and is everywhere accurate to within $pm 0.0052\$ when evaluating the first exponential integral, \$E_1\$. To achieve this result, a methodology that makes use of analytically known limiting behaviors at either extreme of the domain is employed. Because the fit accurately captures limiting behaviors of the \$E_1\$ function, more accuracy is retained when the fit is used as part of the scheme to evaluate higher-order exponential integrals, \$E_n\$, as compared with the use of brute-force fits to \$E_1\$, which fail to accurately model limiting behaviors. Furthermore, because the fit is compact, no special accommodations are required (as in the case of spliced piecewise fits) to smooth the value, slope, and higher derivatives in the transition region between two piecewise domains. The general methodology employed to develop this fit is outlined, since it may be used for other problems as well."

— axiom.bib —

}

```
number = "ARL-TR-1758",
  institution = "U.S. Army Ballistic Research Laboratory,
               Aberdeen Proving Ground, MD",
  year = "1998",
  month = "September",
  paper = "Se09.pdf",
  abstract = "
   This is a writeup detailing the more accurate fits to E_1(x),
   relative to those presented in ARL-TR-1758. My actual fits are to
   [F1 = [x \in x_1(x)]] which spans a functional range from 0 to 1.
   The best accuracy I have been yet able to achieve, defined by limiting
   the value of [[(F1)_{fit} - F1]/F1] over the domain, is
    approximately 3.1E-07 with a 12-parameter fit, which unfortunately
    isn't quite to 32-bit floating-point accuracy. Nonetheless, the fit
    is not a piecewise fit, but rather a single continuous function over
    the domain of nonnegative x, which avoids some of the problems
   associated with piecewise domain splicing."
}
```

2.15 Polynomial GCD

```
— axiom.bib —
```

```
publisher = "Addison-Wesley",
            — axiom.bib —
@article{Ma90,
  author = "Ma, Keju and {von zur Gathen}, Joachim",
  title =
    "Analysis of Euclidean Algorithms for Polynomials over Finite Fields",
  journal = "J. Symbolic Computation",
  year = "1990",
  volume = "9",
  pages = "429-455",
  url = "http://www.researchgate.net/publication/220161718_Analysis_of_Euclidean_Algorithms_for_Polynomials
  paper = "Ma90.pdf",
  abstract = "
   This paper analyzes the Euclidean algorithm and some variants of it
    for computing the greatest common divisor of two univariate polynomials
    over a finite field. The minimum, maximum, and average number of
    arithmetic operations both on polynomials and in the ground field
    are derived."
}
            — ignore —
\bibitem[Naylor 00a]{N00} Naylor, Bill
  title = "Polynomial GCD Using Straight Line Program Representation",
PhD. Thesis, University of Bath, 2000
  url = "http://www.sci.csd.uwo.ca/~bill/thesis.ps",
  paper = "NOO.pdf",
  abstract = "
    This thesis is concerned with calculating polynomial greatest common
```

In the Introduction chapter, we introduce the problem and describe some of the traditional representations for polynomials, we then talk about some of the general subjects central to the thesis, terminating with a synopsis of the category theory which is central to the Axiom computer algebra system used during this research.

divisors using straight line program representation.

The second chapter is devoted to describing category theory. We follow

with a chapter detailing the important sections of computer code written in order to investigate the straight line program subject. The following chapter on evalution strategies and algorithms which are dependant on these follows, the major algorith which is dependant on evaluation and which is central to our theis being that of equality checking. This is indeed central to many mathematical problems. Interpolation, that is the determination of coefficients of a polynomial is the subject of the next chapter. This is very important for many straight line program algorithms, as their non-canonical structure implies that it is relatively difficult to determine coefficients, these being the basic objects that many algorithms work on. We talk about three separate interpolation techniques and compare their advantages and disadvantages. The final two chapters describe some of the results we have obtained from this research and finally conclusions we have drawn as to the viability of the straight line program approach and possible extensions.

Finally we terminate with a number of appendices discussing side subjects encountered during the thesis."

— ignore —

\bibitem[Shoup 93]{ST-PGCD-Sh93} Shoup, Victor
 title = "Factoring Polynomials over Finite Fields: Asymptotic Complexity vs Reality*",
Proc. IMACS Symposium, Lille, France, (1993)
 url = "http://www.shoup.net/papers/lille.pdf",
 paper = "ST-PGCD-Sh93.pdf",
 abstract = "
This paper compares the elevations by Papelsham Contact and

This paper compares the algorithms by Berlekamp, Cantor and Zassenhaus, and Gathen and Shoup to conclude that (a) if large polynomials are factored the FFT should be used for polynomial multiplication and division, (b) Gathen and Shoup should be used if the number of irreducible factors of \$f\$ is small. (c) if nothing is know about the degrees of the factors then Berlekamp's algorithm should be used."

— ignore —

\bibitem[Gathen 01]{ST-PGCD-Ga01} von zur Gathen, Joachim; Panario, Daniel
 title = "Factoring Polynomials Over Finite Fields: A Survey",
J. Symbolic Computation (2001) Vol 31, pp3-17\hfill{}
 url =

"http://people.csail.mit.edu/dmoshdov/courses/codes/poly-factorization.pdf",
paper = "ST-PGCD-GaO1.pdf",
keywords = "survey",
abstract = "

This survey reviews several algorithms for the factorization of
univariate polynomials over finite fields. We emphasize the main ideas
of the methods and provide and up-to-date bibliography of the problem.
This paper gives algorithms for {\sl squarefree factorization},
{\sl distinct-degree factorization}, and {\sl equal-degree factorization}.
The first and second algorithms are deterministic, the third is
probabilistic."

— ignore —

\bibitem[van Hoeij]{Hoeij04} {van Hoeij}, Mark; Monagan, Michael
 title = "Algorithms for Polynomial GCD Computation over Algebraic Function Fields",
 url = "http://www.cecm.sfu.ca/personal/mmonagan/papers/AFGCD.pdf",
 paper = "Hoeij04.pdf",
 abstract = "

Let L be an algebraic function field in $k \ge 0$ parameters t_1,\ldots,k . Let f_1 , f_2 be non-zero polynomials in L[x]. We give two algorithms for computing their gcd. The first, a modular GCD algorithm, is an extension of the modular GCD algorithm for Brown for L for L function fields. The second, a fraction-free algorithm, is a modification of the Moreno Maza and Rioboo algorithm for computing gcds over triangular sets. The modification reduces coefficient grownth in L to be linear. We give an empirical comparison of the two algorithms using implementations in Maple."

— ignore —

\bibitem[Wang 78]{Wang78} Wang, Paul S.
title = "An Improved Multivariate Polynomial Factoring Algorithm",
Mathematics of Computation, Vol 32, No 144 Oct 1978, pp1215-1231

url = "http://www.ams.org/journals/mcom/1978-32-144/S0025-5718-1978-0568284-3/S0025-5718-1978-0568284-3.p. paper = "Wang78.pdf",

bstract = "

A new algorithm for factoring multivariate polynomials over the integers based on an algorithm by Wang and Rothschild is described. The new algorithm has improved strategies for dealing with the known problems of the original algorithm, namely, the leading coefficient

problem, the bad-zero problem and the occurence of extraneous factors. It has an algorithm for correctly predetermining leading coefficients of the factors. A new and efficient p-adic algorith named EEZ is described. Basically it is a linearly convergent variable-by-variable parallel construction. The improved algorithm is generally faster and requires less store than the original algorithm. Machine examples with comparative timing are included."

— ignore —

\bibitem[Wiki 4]{Wiki4}.

title = "Polynomial greatest common divisor",

url = "http://en.wikipedia.org/wiki/Polynomial_greatest_common_divisor",

2.16 Category Theory

- ignore -

\bibitem[Baez 09]{Baez09} Baez, John C.; Stay, Mike
 title = "Physics, Topology, Logic and Computation: A Rosetta Stone",
 url = "http://arxiv.org/pdf/0903.0340v3.pdf",
 paper = "Baez09.pdf",
 abstract = "

In physics, Feynman diagrams are used to reason about quantum processes. In the 1980s, it became clear that underlying these diagrams is a powerful analogy between quantum physics and topology. Namely, a linear operator behaves very much like a ''cobordism'': a manifold representing spacetime, going between two manifolds representing space. But this was just the beginning: simiar diagrams can be used to reason about logic, where they represent proofs, and computation, where they represent programs. With the rise of interest in quantum cryptography and quantum computation, it became clear that there is an extensive network of analogies between physics, topology, logic and computation. In this expository paper, we make some of these analogies precise using the concept of ''closed symmetric monodial category''. We assume no prior knowledge of category theory, proof theory or computer science."

— ignore —

\bibitem[Meijer 91]{Meij91} Meijer, Erik; Fokkinga, Maarten; Paterson, Ross
 title = "Functional Programming with Bananas, Lenses, Envelopes and Barbed Wire",
 url = "http://eprints.eemcs.utwente.nl/7281/01/db-utwente-40501F46.pdf",
 paper = "Meij91.pdf",
 abstract = "

We develop a calculus for lazy functional programming based on recursion operators associated with data type definitions. For these operators we derive various algebraic laws that are useful in deriving and manipulating programs. We shall show that all example functions in Bird and Wadler's ''Introduction to Functional Programming'' can be expressed using these operators."

— ignore —

\bibitem[Youssef 04]{You04} Youssef, Saul
 title = "Prospects for Category Theory in Aldor",
 year = "2004",
 paper = "You04.pdf",
 abstract = "

Ways of encorporating category theory constructions and results into the Aldor language are discussed. The main features of Aldor which make this possible are identified, examples of categorical constructions are provided and a suggestion is made for a foundation for rigorous results."

2.17 Proving Axiom Correct

— ignore —

 aided formal reasoning to computer algebra, and argue that embedded verification support allows users to enjoy its benefits without wrestling with technicalities. We illustrate this claim by considering symbolic definite integration, and present a verifiable symbolic definite integral table look up: a system which matches a query comprising a definite integral with parameters and side conditions, against an entry in a verifiable table and uses a call to a library of lemmas about the reals in the theorem prover PVS to aid in the transformation of the table entry into an answer. We present the full model of such a system as well as a description of our prototype implementation showing the efficacy of such a system: for example, the prototype is able to obtain correct answers in cases where computer algebra systems [CAS] do not. We extend upon Fateman's web-based table by including parametric limits of integration and queries with side conditions."

- ignore -

\bibitem[Adams 01]{Adam01} Adams, Andrew; Dunstan, Martin; Gottliebsen, Hanne; Kelsey, Tom; Martin, Ursula; Owre, Sam

url = "http://www.csl.sri.com/~owre/papers/tphols01/tphols01.pdf",
paper = "Adam01.pdf",
abstract = "

We describe an interface between version 6 of the Maple computer algebra system with the PVS automated theorem prover. The interface is designed to allow Maple users access to the robust and checkable proof environment of PVS. We also extend this environment by the provision of a library of proof strategies for use in real analysis. We demonstrate examples using the interface and the real analysis library. These examples provide proofs which are both illustrative and applicable to genuine symbolic computation problems."

— axiom.bib —

@article{Bres93,

author = "Bressoud, David",
title = "Review of The problems of mathematics",
journal = "Math. Intell.",
volume = "15",
number = "4",

```
year = "1993",
 pages = "71-73"
            — axiom.bib —
@article{Bulo04,
  author = "Medina-Bulo, I. and Palomo-Lozano, F. and Alonso-Jim\'enez, J.A.
          and Ruiz-Reina, J.L.",
 title = "Verified Computer Algebra in ACL2",
  journal = "ASIC 2004, LNAI 3249",
 year = "2004",
 pages = "171-184",
 paper = "Bulo04.pdf",
  abstract = "In this paper, we present the formal verification of a
   Common Lisp implementation of Buchberger's algorithm for computing
   Groebner bases of polynomial ideals. This work is carried out in the
   ACL2 system and shows how verified Computer Algebra can be achieved
   in an executable logic."
}
           — axiom.bib —
@book{Chli15,
  author = "Chlipala, Adam",
 title = "Certified Programming with Dependent Types",
 year = "2015",
 url = "http://adam.chlipala.net/cpdt/cpdt.pdf",
 publisher = "MIT Press",
 isbn = "9780262026659",
 paper = "Chli15.pdf"
            — axiom.bib —
@article{Mahb06,
  author = "Mahboubi, Assia",
```

— axiom.bib —

@misc{Pier15,

author = "Pierce, Benjamin C. and Casinghino, Chris and Gaboardi, Marco and Greenberg, Michael and Hritcu, Catalin and Sjoberg, Vilhelm and Yorgey, Brent",

title = "Software Foundations",
year = "2015",
file = "Pier15.tgz",
abstract =

"This electronic book is a course on Software Foundations, the mathematical underpinnings of reliable software. Topics include basic concepts of logic, computer-assisted theorem proving, the Coq proof assistant, functional programming, operational semantics, Hoare logic, and static type systems. The exposition is intended for a broad range of readers, from advanced undergraduates to PhD students and researchers. No specific background in logic or programming languages is assumed, though a degree of mathematical maturity will be helpful.

The principal novelty of the course is that it is one hundred per cent formalized and machine-checked: the entire text is literally a script for Coq. It is intended to be read alongside an interactive session with Coq. All the details in the text are fully formalized in Coq, and the exercises are designed to be worked using Coq.

The files are organized into a sequence of core chapters, covering about one semester's worth of material and organized into a coherent linear narrative, plus a number of appendices covering additional topics. All the core chapters are suitable for both upper-level undergraduate and graduate students."

}

— axiom.bib —

```
@article{Ther01,
   author = "Th\'ery, Laurent",
   title = "A Machine-Checked Implementation of Buchberger's Algorithm",
   journal = "Journal of Automated Reasoning",
   volume = "26",
   year = "2001",
   pages = "107-137",
   paper = "Ther01.pdf",
   abstract = "We present an implementation of Buchberger's algorithm that
    has been proved correct within the proof assistant Coq. The
   implementation contains the basic algorithm plus two standard
   optimizations."
}
```

— ignore —

The use of computer algebra is usually considered beneficial for mechanised reasoning in mathematical domains. We present a case study, in the application domain of coding theory, that supports this claim: the mechanised proofs depend on non-trivial algorithms from computer algebra and increase the reasoning power of the theorem prover.

The unsoundness of computer algebra systems is a major problem in interfacing them to theorem provers. Our approach to obtaining a sound overall system is not blanket distrust but based on the distinction between algorithms we call sound and {\sl ad hoc} respectively. This distinction is blurred in most computer algebra systems. Our experimental interface therefore uses a computer algebra library. It is based on formal specifications for the algorithms, and links the computer algebra library Sumit to the prover Isabelle.

We give details of the interface, the use of the computer algebra system on the tactic-level of Isabelle and its integration into proof

procedures."

— ignore —

\bibitem[Bertot 04]{Bert04} Bertot, Yves; Cast\'eran, Pierre
 title = "Interactive Theorem Proving and Program Development",
 isbn = "3-540-20854-2",
 abstract = "

Coq is an interactive proof assistant for the development of mathematical theories and formally certified software. It is based on a theory called the calculus of inductive constructions, a variant of type theory.

This book provides a pragmatic introduction to the development of proofs and certified programs using Coq. With its large collection of examples and exercies it is an invaluable tool for researchers, students, and engineers interested in formal methods and the development of zero-fault software."

— ignore —

Abstraction is a powerful tool for developers and it is offered by numerous features such as polymorphism, classes, modules, and functors, \$\ldots\$ A working programmer may be confused by this abundance. We develop a computer algebra library which is being certificed. Reporting this experience made with a language (Ocaml) offering all these features, we argue that the are all needed together. We compare several ways of using classes to represent algebraic concepts, trying to follow as close as possible mathematical specification. Thenwe show how to combine classes and modules to produce code having very strong typing properties. Currently, this library is made of one hundred units of functional code and behaves faster than analogous ones such as Axiom."

— ignore —

\bibitem[Boulme 01]{BHHMR01}
Boulm\'e, S.; Hardin, T.; Hirschkoff, D.; M\'enissier-Morain, V.; Rioboo, R.
 title = "On the way to certify Computer Algebra Systems",
 year = "2001",

Calculemus-2001

paper = "BHHMR01.pdf",

abstract = "

The FOC project aims at supporting, within a coherent software system, the entire process of mathematical computation, starting with proved theories, ending with certified implementations of algorithms. In this paper, we explain our design requirements for the implementation, using polynomials as a running example. Indeed, proving correctness of implementations depends heavily on the way this design allows mathematical properties to be truly handled at the programming level.

The FOC project, started at the fall of 1997, is aimed to build a programming environment for the development of certified symbolic computation. The working languages are Coq and Ocaml. In this paper, we present first the motivations of the project. We then explain why and how our concern for proving properties of programs has led us to certain implementation choices in Ocaml. This way, the sources express exactly the mathematical dependencies between different structures. This may ease the achievement of proofs."

— ignore —

\bibitem[Daly 10]{Daly10} Daly, Timothy

title = "Intel Instruction Semantics Generator",

url = "http://daly.axiom-developer.org/TimothyDaly_files/publications/sei/intel/intel.pdf",
paper = "Daly10.pdf",

abstract = "

Given an Intel x86 binary, extract the semantics of the instruction stream as Conditional Concurrent Assignments (CCAs). These CCAs represent the semantics of each individual instruction. They can be composed to represent higher level semantics."

— ignore —

\bibitem[Danielsson 06]{Dani06} Danielsson, Nils Anders; Hughes, John;

Jansson, Patrik; Gibbons, Jeremy
 title = "Fast and Loose Reasoning is Morally Correct",
 year = "2005",
ACM POPL'06 January 2006, Charleston, South Carolina, USA
 paper = "Dani06.pdf",

abstract = "

Functional programmers often reason about programs as if they were written in a total language, expecting the results to carry over to non-toal (partial) languages. We justify such reasoning.

Two languages are defined, one total and one partial, with identical syntax. The semantics of the partial language includes partial and infinite values, and all types are lifted, including the function spaces. A partial equivalence relation (PER) is then defined, the domain of which is the total subset of the partial language. For types not containing function spaces the PER relates equal values, and functions are related if they map related values to related values.

It is proved that if two closed terms have the same semantics in the total language, then they have related semantics in the partial language. It is also shown that the PER gives rise to a bicartesian closed category which can be used to reason about values in the domain of the relation."

— ignore —

\bibitem[Davenport 12]{Davenp12} Davenport, James H.; Bradford, Russell; England, Matthew; Wilson, David

url = "http://arxiv.org/pdf/1212.5417.pdf",
paper = "Davenp12.pdf",

abstract = "

In considering the reliability of numerical programs, it is normal to "'limit our study to the semantics dealing with numerical precision". On the other hand, there is a great deal of work on the reliability of programs that essentially ignores the numerics. The thesis of this paper is that there is a class of problems that fall between these two, which could be described as "does the low-level arithmetic implement the high-level mathematics". Many of these problems arise because mathematics, particularly the mathematics of the complex numbers, is more difficult than expected: for example the complex function log is not continuous, writing down a program to compute an inverse function is more complicated than just solving an equation, and many algebraic simplification rules are not universally valid.

The good news is that these problems are {\sl theoretically} capable of being solved, and are {\sl practically} close to being solved, but not yet solved, in several real-world examples. However, there is still a long way to go before implementations match the theoretical possibilities."

— ignore —

\bibitem[Dolzmann 97]{Dolz97} Dolzmann, Andreas; Sturm, Thomas
 title = "Guarded Expressions in Practice",
 url = "http://redlog.dolzmann.de/papers/pdf/MIP-9702.pdf",
 paper = "Dolz97.pdf",
 abstract = "

Computer algebra systems typically drop some degenerate cases when evaluating expressions, e.g. \$x/x\$ becomes 1 dropping the case \$x=0\$. We claim that it is feasible in practice to compute also the degenerate cases yielding {\sl guarded expressions}. We work over real closed fields but our ideas about handling guarded expressions can be easily transferred to other situations. Using formulas as guards provides a powerful tool for heuristically reducing the combinatorial explosion of cases: equivalent, redundant, tautological, and contradictive cases can be detected by simplification and quantifier elimination. Our approach allows to simplify the expressions on the basis of simplification knowledge on the logical side. The method described in this paper is implemented in the REDUCE package GUARDIAN, which is freely available on the WWW."

— ignore —

\bibitem[Dos Reis 11]{DR11} Dos Reis, Gabriel; Matthews, David; Li, Yue
 title = "Retargeting OpenAxiom to Poly/ML: Towards an Integrated Proof Assistants and Computer Algebra Sy.
Calculemus (2011) Springer

url = "http://paradise.caltech.edu/~yli/paper/oa-polyml.pdf",
keywords = "axiomref",
paper = "DR11.pdf",
abstract = "

This paper presents an ongoing effort to integrate the Axiom family of computer algebra systems with Poly/ML-based proof assistants in the same framework. A long term goal is to make a large set of efficient implementations of algebraic algorithms available to popular proof assistants, and also to bring the power of mechanized formal verification to a family of strongly typed computer algebra systems at

a modest cost. Our approach is based on retargeting the code generator of the OpenAxiom compiler to the Poly/ML abstract machine."

— ignore —

\bibitem[Dunstan 00a]{Dun00a} Dunstan, Martin N.
 title = "Adding Larch/Aldor Specifications to Aldor",
 paper = "Dunxx.pdf",
 abstract = "

We describe a proposal to add Larch-style annotations to the Aldor programming language, based on our PhD research. The annotations are intended to be machine-checkable and may be used for a variety of purposes ranging from compiler optimizations to verification condition (VC) generation. In this report we highlight the options available and describe the changes which would need to be made to the compiler to make use of this technology."

— ignore —

\bibitem[Dunstan 98]{Dun98} Dunstan, Martin; Kelsey, Tom; Linton, Steve; Martin, Ursula

title = "Lightweight Formal Methods For Computer Algebra Systems",
url = "http://www.cs.st-andrews.ac.uk/~tom/pub/issac98.pdf",
paper = "Dun98.pdf",
keywords = "axiomref",
abstract = "

Demonstrates the use of formal methods tools to provide a semantics for the type hierarchy of the Axiom computer algebra system, and a methodology for Aldor program analysis and verification. There are examples of abstract specifications of Axiom primitives."

- ignore -

\bibitem[Dunstan 99a]{Dun99a} Dunstan, MN
 title = "Larch/Aldor - A Larch BISL for AXIOM and Aldor",
 year = "1999",
PhD Thesis, 1999

```
url = "http://www.cs.st-andrews.uk/files/publications/Dun99.php",
paper = "Dun99a.pdf",
keywords = "axiomref",
abstract = "
```

In this thesis we investigate the use of lightweight formal methods and verification conditions (VCs) to help improve the reliability of components constructed within a computer algebra system. We follow the Larch approach to formal methods and have designed a new behavioural interface specification language (BISL) for use with Aldor: the compiled extension language of Axiom and a fully-featured programming language in its own right. We describe our idea of lightweight formal methods, present a design for a lightweight verification condition generator and review our implementation of a prototype verification condition generator for Larch/Aldor."

— ignore —

```
\bibitem[Dunstan 00]{Dun00} Dunstan, Martin; Kelsey, Tom; Martin, Ursula;
Linton, Steve
  title = "Formal Methods for Extensions to CAS",
FME'99, Toulouse, France, Sept 20-24, 1999, pp 1758-1777
  url = "http://tom.host.cs.st-andrews.ac.uk/pub/fm99.ps",
  paper = "Dun00.pdf",
  abstract = "
```

We demonstrate the use of formal methods tools to provide a semantics for the type hierarchy of the AXIOM computer algebra system, and a methodology for Aldor program analysis and verification. We give a case study of abstract specifications of AXIOM primitives, and provide an interface between these abstractions and Aldor code."

— axiom.bib —

```
@misc{Hard13,
```

```
author = "Hardin, David S. and McClurg, Jedidiah R. and Davis, Jennifer A.",
title = "Creating Formally Verified Components for Layered Assurance with an LLVM to ACL2 Translator",
url = "http://www.jrmcclurg.com/papers/law_2013_paper.pdf",
paper = "Hard13.pdf",
abstract = "
```

This paper describes an effort to create a library of formally verified software component models from code that have been compiled using the Low-Level Virtual Machine (LLVM) intermediate form. The idea is to build a translator from LLVM to the applicative subset of Common

}

Lisp accepted by the ACL2 theorem prover. They perform verification of the component model using ACL2's automated reasoning capabilities."

— axiom.bib —

@misc{Hard14,

In our current work a library of formally verified software components is to be created, and assembled, using the Low-Level Virtual Machine (LLVM) intermediate form, into subsystems whose top-level assurance relies on the assurance of the individual components. We have thus undertaken a project to build a translator from LLVM to the applicative subset of Common Lisp accepted by the ACL2 theorem prover. Our translator produces executable ACL2 formal models, allowing us to both prove theorems about the translated models as well as validate those models by testing. The resulting models can be translated and certified without user intervention, even for code with loops, thanks to the use of the def::ung macro which allows us to defer the question of termination. Initial measurements of concrete execution for translated LLVM functions indicate that performance is nearly 2.4 million LLVM instructions per second on a typical laptop computer. In this paper we overview the translation process and illustrate the translator's capabilities by way of a concrete example, including both a functional correctness theorem as well as a validation test for that example."

— axiom.bib —

@book{Lamp02,

```
author = "Lamport, Leslie",
title = "Specifying Systems",
year = "2002",
url = "http://research.microsoft.com/en-us/um/people/lamport/tla/book-02-08-08.pdf",
publisher = "Addison-Wesley",
isbn = "0-321-14306-X",
```

```
paper = "Lamp02.pdf",
            — axiom.bib —
@misc{Lamp13,
  author = "Lamport, Leslie",
 title = "Errata to Specifying Systems",
  year = "2013",
  url = "http://research.microsoft.com/en-us/um/people/lamport/tla/errata-1.pdf",
  publisher = "Microsoft",
  paper = "Lamp13.pdf",
  abstract = "
    These are all the errors and omissions to the first printing (July
    2002) of the book {\sl Specifying Systems} reported as of 29 October
    2013. Positions in the book are indicated by page and line number,
    where the top line of a page is number 1 and the bottom line is number
    $-1$. A running head and a page number are not considered to be lines,
    but all other lines are. Please report any additional errors to the
    author, whose email address is posted on { htt http://lamport.org}. The
    first person to report an error will be acknowledged in any revised
    edition."
}
            — axiom.bib —
@misc{Lamp14,
  author = "Lamport, Leslie",
 title = "How to Write a $21^{st}$ Century Proof",
 year = "2014",
  url = "http://research.microsoft.com/en-us/um/people/lamport/pubs/paper.pdf",
  publisher = "Microsoft",
  paper = "Lamp14.pdf",
  abstract = "
   A method of writing proofs is described that makes it harder to prove
   things that are not true. The method, based on hierarchical
   structuring, is simple and practical. The author's twenty years of
   experience writing such proofs is discussed."
}
```

— axiom.bib —

```
@misc{Lamp14a,
   author = "Lamport, Leslie",
   title = "Talk: How to Write a $21^{st}$ Century Proof",
   year = "2014",
   url =
"http://hits.mediasite.com/mediasite/Play/29d825439b3c49f088d35555426fbdf81d",
   comment = "2nd Heidelberg Laureate Forum Lecture Tuesday Sep 23, 2014"
}
```

— ignore —

```
\bibitem[Martin 97]{Mart97} Martin, U.; Shand, D.
```

```
title = "Investigating some Embedded Verification Techniques for Computer Algebra Systems",
url = "http://www.risc.jku.at/conferences/Theorema/papers/shand.ps.gz",
paper = "Mart97.ps",
abstract = "
```

This paper reports some preliminary ideas on a collaborative project between St. Andrews University in the UK and NAG Ltd. The project aims to use embedded verification techniques to improve the reliability and mathematical soundness of computer algebra systems. We give some history of attempts to integrate computer algebra systems and automated theorem provers and discuss possible advantages and disadvantages of these approaches. We also discuss some possible case studies."

— axiom.bib —

```
@book{Maso86,
  author = "Mason, Ian A.",
  title = "The Semantics of Destructive Lisp",
  publisher = "Center for the Study of Language and Information",
  year = "1986",
  isbn = "0-937073-06-7",
  abstract = "
```

Our basic premise is that the ability to construct and modify programs will not improve without a new and comprehensive look at the entire programming process. Past theoretical research, say, in the logic of programs, has tended to focus on methods for reasoning about

individual programs; little has been done, it seems to us, to develop a sound understanding of the process of programming -- the process by which programs evolve in concept and in practice. At present, we lack the means to describe the techniques of program construction and improvement in ways that properly link verification, documentation and adaptability."

— ignore —

}

\bibitem[Newcombe 13]{Newc13} Newcombe, Chris; Rath, Tim; Zhang, Fan;
Munteanu, Bogdan; Brooker, Marc; Deardeuff, Michael
 title = "Use of Formal Methods at Amazon Web Services",
 url = "http://research.microsoft.com/en-us/um/people/lamport/tla/formal-methods-amazon.pdf",
 abstract = "

In order to find subtle bugs in a system design, it is necessary to have a precise description of that design. There are at least two major benefits to writing a precise design; the author is forced to think more clearly, which helps eliminate "'plausible hand-waving", and tools can be applied to check for errors in the design, even while it is being written. In contrast, conventional design documents consist of prose, static diagrams, and perhaps pseudo-code in an ad hoc untestable language. Such descriptions are far from precise; they are often ambiguous, or omit critical aspects such as partial failure or the granularity of concurrency (i.e. which constructs are assumed to be atomic). At the other end of the spectrum, the final executable code is unambiguous, but contains an overwhelming amount of detail. We needed to be able to capture the essence of a design in a few hundred lines of precise description. As our designs are unavoidably complex, we need a highly-expressive language, far above the level of code, but with precise semantics. That expressivity must cover real-world concurrency and fault-tolerance. And, as we wish to build services quickly, we wanted a language that is simple to learn and apply, avoiding esoteric concepts. We also very much wanted an existing ecosystem of tools. We found what we were looking for in TLA+, a formal specification language."

— ignore —

\bibitem[Poll 99a]{P99a} Poll, Erik
 title = "The Type System of Axiom",
 url = "http://www.cs.ru.nl/E.Poll/talks/axiom.pdf",

```
paper = "P99a.pdf",
abstract = "
```

This is a slide deck from a talk on the correspondence between Axiom/Aldor types and Logic."

— ignore —

```
\bibitem[Poll 99]{PT99} Poll, Erik; Thompson, Simon
 title = "The Type System of Aldor",
 url = "http://www.cs.kent.ac.uk/pubs/1999/874/content.ps",
 paper = "PT99.pdf",
 abstract = "
```

This paper gives a formal description of $\operatorname{--}$ at least a part of $\operatorname{--}$ the type system of Aldor, the extension language of the Axiom. In the process of doing this a critique of the design of the system $\ \ \,$ emerges."

— ignore —

\bibitem[Poll (a)]{PTxx} Poll, Erik; Thompson, Simon

title = "Adding the axioms to Axiom. Toward a system of automated reasoning in Aldor", url = "http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.7.1457&rep=rep1&type=ps", paper = "PTxx.pdf",

keywords = "axiomref",

abstract = "

This paper examines the proposal of using the type system of Axiom to represent a logic, and thus to use the constructions of Axiom to handle the logic and represent proofs and propositions, in the same way as is done in theorem provers based on type theory such as Nuprl or Coq.

The paper shows an interesting way to decorate Axiom with pre- and post-conditions.

The Curry-Howard correspondence used is

\begin{verbatim}

PROGRAMMING LOGIC Туре Formula Program Proof (\ldots,\ldots) Conjunction Product/record type Sum/union type \/ Disjunction -> Function type Implication

```
Dependent function type (x:A) \rightarrow B(x) Universal quantifier
    Dependent product type (x:A,B(x)) Existential quantifier
    Empty type
                           Exit
                                          Contradictory proposition
    One element type
                           Triv
                                         True proposition
    \end{verbatim}"
            — ignore —
\bibitem[Poll 00]{PT00} Poll, Erik; Thompson, Simon
  title = "Integrating Computer Algebra and Reasoning through the Type System of Aldor",
  paper = "PT00.pdf",
  keywords = "axiomref",
  abstract = "
    A number of combinations of reasoning and computer algebra systems
    have been proposed; in this paper we describe another, namely a way to
    incorporate a logic in the computer algebra system Axiom. We examine
    the type system of Aldor -- the Axiom Library Compiler -- and show
    that with some modifications we can use the dependent types of the
    system to model a logic, under the Curry-Howeard isomorphism. We give
    a number of example applications of the logi we construct and explain
    a prototype implementation of a modified type-checking system written
    in Haskell."
            — axiom.bib —
@misc{Robe15,
author = "Roberts, Siobhan",
title = "In Mathematics, Mistakes Aren't What They Used To Be",
year = 2015,
url = "http://nautil.us/issue/24/error/In-mathematics-mistakes-arent-what-they-used-to-be"
```

2.18 Interval Arithmetic

— ignore —

```
\bibitem[Boehm 86]{Boe86} Boehm, Hans-J.; Cartwright, Robert; Riggle, Mark;
O'Donnell, Michael J.
  title = "Exact Real Arithmetic: A Case Study in Higher Order Programming",
  url = "http://dev.acm.org/pubs/citations/proceedings/lfp/319838/p162-boehm",
  paper = "Boe86.pdf",
            — ignore —
\bibitem[Briggs 04]{Bri04} Briggs, Keith
  title = "Exact real arithmetic",
  url = "http://keithbriggs.info/documents/xr-kent-talk-pp.pdf",
 paper = "Bri04.pdf",
            — ignore —
\bibitem[Fateman 94]{Fat94} Fateman, Richard J.; Yan, Tak W.
  title = "Computation with the Extended Rational Numbers and an Application to Interval Arithme
  url = "http://www.cs.berkeley.edu/~fateman/papers/extrat.pdf",
  paper = "Fat94.pdf",
  abstract = "
    Programming languages such as Common Lisp, and virtually every
    computer algebra system (CAS), support exact arbitrary-precision
    integer arithmetic as well as exect rational number computation.
    Several CAS include interval arithmetic directly, but not in the
    extended form indicated here. We explain why changes to the usual
   rational number system to include infinity and ''not-a-number'' may be
    useful, especially to support robust interval computation. We describe
    techniques for implementing these changes."
            — axiom.bib —
@incollection{Lamb06,
  author = "Lambov, Branimir",
  title = "Interval Arithmetic Using SSE-2",
```

booktitle = "Lecture Notes in Computer Science",

publisher = "Springer-Verlag",

year = "2006",

2.19. NUMERICS 65

```
isbn = "978-3-540-85520-0",
pages = "102-113"
}
```

2.19 Numerics

— ignore —

\bibitem[Atkinson 09]{Atk09} Atkinson, Kendall; Han, Welmin; Stewear, David
 title = "Numerical Solution of Ordinary Differential Equations",
 url = "http://homepage.math.uiowa.edu/~atkinson/papers/NAODE_Book.pdf",
 paper = "Atk09.pdf",
 abstract = "

This book is an expanded version of supplementary notes that we used for a course on ordinary differential equations for upper-division undergraduate students and beginning graduate students in mathematics, engineering, and sciences. The book introduces the numerical analysis of differential equations, describing the mathematical background for understanding numerical methods and giving information on what to expect when using them. As a reason for studying numerical methods as a part of a more general course on differential equations, many of the basic ideas of the numerical analysis of differential equations are tied closely to theoretical behavior associated with the problem being solved. For example, the criteria for the stability of a numerical method is closely connected to the stability of the differential equation problem being solved."

— ignore —

\bibitem[Crank 96]{Cran96} Crank, J.; Nicolson, P.

title = "A practical method for numerical evaluations of solutions of partial differential equations of hadvances in Computational Mathematics Vol 6 pp207-226 (1996)

url = "http://www.acms.arizona.edu/FemtoTheory/MK_personal/opti547/literature/CNMethod-original.pdf",
paper = "Cran96.pdf",

```
\bibitem[Lef\'evre 06]{Lef06} Lef\'evre, Vincent; Stehl\'e, Damien;
  title = "Worst Cases for the Exponential Function in the IEEE-754r decimal64 Format",
in Lecture Notes in Computer Science, Springer ISBN 978-3-540-85520-0
(2006) pp114-125
  abstract = "
    We searched for the worst cases for correct rounding of the
    exponential function in the IEEE 754r decimal64 format, and computed
    all the bad cases whose distance from a breakpoint (for all rounding
   modes) is less than 10^{-15} ulp, and we give the worst ones. In
    particular, the worst case for
    \displaystyle {\text{yert}} \propto 3 \times 10^{-11} is
    \exp(9.407822313572878x10^{-2}) =
    1.0986456820663385000000000000000278 \verb|\ldots|
    This work can be extended to other elementary functions in the decimal64
    format and allows the design of reasonably fast routines that will
    evaluate these functions with correct rounding, at least in some
    situations."
            — axiom.bib —
@book{Hamm62,
  author = "Hamming, R W.",
  title = "Numerical Methods for Scientists and Engineers",
  publisher = "Dover",
 year = "1973",
 isbn = "0-486-65241-6"
```

2.20 Advanced Documentation

```
\bibitem [Bostock 14]{Bos14} Bostock, Mike
  title = "Visualizing Algorithms",
  url = "http://bost.ocks.org/mike/algorithms",
  abstract = "
   This website hosts various ways of visualizing algorithms. The hope is
```

that these kind of techniques can be applied to Axiom."

— axiom.bib —

@misc{Kama15,

title = "Computerising Mathematical Text",

year = "2015",

abstract =

"Mathematical texts can be computerised in many ways that capture differing amounts of the mathematical meaning. At one end, there is document imaging, which captures the arrangement of black marks on paper, while at the other end there are proof assistants (e.g. Mizar, Isabelle, Coq, etc.), which capture the full mathematical meaning and have proofs expressed in a formal foundation of mathematics. In between, there are computer typesetting systems (e.g. Latex and Presentation MathML) and semantically oriented systems (e.g. Content MathML, OpenMath, OMDoc, etc.). In this paper we advocate a style of computerisation of mathematical texts which is flexible enough to connect the different approaches to computerisation, which allows various degrees of formalsation, and which is compatible with different logical frameworks (e.g. set theory, category theory, type theory, etc.) and proof systems. The basic idea is to allow a man-machine collaboration which weaves human input with machine computation at every step in the way. We propose that the huge step from informal mathematics to fully formalised mathematics be divided into smaller steps, each of which is a fully developed method in which human input is minimal."

}

— ignore —

title = "Representation of mathematical object in interactive books",
paper = "Leexx.pdf",

abstract =

We present a model for the representation of mathematical objects in structured electronic documents, in a way that allows for interaction with applications such as computer algebra systems and proof checkers. Using a representation that reflects only the intrinsic information of an object, and storing application-dependent information in so-called {\sl application descriptions}, it is shown how the translation from the internal to an external representation and {\sl vice versa} can be achieved. Hereby a formalisation of the concept of {\sl context} is introduced. The proposed scheme allows for a high degree of application integration, e.g., parallel evaluation of subexpressions (by different computer algebra systems), or a proof checker using a computer algebra system to verify an equation involving a symbolic computation."

— ignore —

\bibitem[Soiffer 91]{Soiff91} Soiffer, Neil Morrell

title = "The Design of a User Interface for Computer Algebra Systems",
url = "http://www.eecs.berkeley.edu/Pubs/TechRpts/1991/CSD-91-626.pdf",
paper = "Soif91.pdf",
abstract = "

This thesis discusses the design and implementation of natural user interfaces for Computer Algebra Systems. Such an interface must not only display expressions generated by the Computer Algebra System in standard mathematical notation, but must also allow easy manipulation and entry of expressions in that notation. The user interface should also assist in understanding of large expressions that are generated by Computer Algebra Systems and should be able to accommodate new notational forms."

— ignore —

\bibitem[Victor 11]{Vict11} Victor, Bret
 title = "Up and Down the Ladder of Abstraction",
 url = "http://worrydream.com/LadderOfAbstraction",
 abstract = "

This interactive essay presents the ladder of abstraction, a technique for thinking explicitly about these levels, so a designer can move among them consciously and confidently. "

```
\bibitem[Victor 12]{Vict12} Victor, Bret
  title = "Inventing on Principle",
  url = "http://www.youtube.com/watch?v=PUv66718DII",
  abstract = "
   This video raises the level of discussion about human-computer
  interaction from a technical question to a question of effectively
  capturing ideas. In particular, this applies well to Axiom's focus on
  literate programming."
```

2.21 Differential Equations

```
— axiom.bib —
@InProceedings{Kalt84,
  author = "Kaltofen, E.",
 title = "A Note on the {Risch} Differential Equation",
 booktitle = "Proc. EUROSAM '84",
 pages = "359--366",
 crossref = "EUROSAM84",
 year = "1984",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/84/Ka84_risch.ps.gz",
 paper = "Kalt84.ps",
            — ignore —
\bibitem[Abramov 95]{Abra95} Abramov, Sergei A.; Bronstein, Manuel;
Petkovsek, Marko
 title = "On Polynomial Solutions of Linear Operator Equations",
   "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
 paper = "Abra95.pdf",
            — ignore —
```

\bibitem[Abramov 01]{Abra01} Abramov, Sergei; Bronstein, Manuel

title = "On Solutions of Linear Functional Systems",
url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
paper = "Abra01.pdf",
abstract = "

We describe a new direct algorithm for transforming a linear system of recurrences into an equivalent one with nonsingular leading or trailing matrix. Our algorithm, which is an improvement to the EG elimination method, uses only elementary linear algebra operations (ranks, kernels, and determinants) to produce an equation satisfied by the degress of the solutions with finite support. As a consequence, we can boudn and compute the polynomial and rational solutions of very general linear functional systems such as systems of differential or (\$q\$)-difference equations."

— ignore —

\bibitem[Bronstein 96b]{Bro96b} Bronstein, Manuel

title = "On the Factorization of Linear Ordinary Differential Operators", Mathematics and Computers in Simulation 42 pp 387-389 (1996)

paper = "Bro96b.pdf",

abstract = "

After reviewing the arithmetic of linear ordinary differential operators, we describe the current status of the factorisation algorithm, specially with respect to factoring over non-algebraically closed constant fields. We also describe recent results from Singer and Ulmer that reduce determining the differential Galois group of an operator to factoring."

— ignore —

\bibitem[Bronstein 96a]{Bro96a} Bronstein, Manuel; Petkovsek, Marko
 title = "An introduction to pseudo-linear algebra",
Theoretical Computer Science V157 pp3-33 (1966)

url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html", paper = "Bro96a.pdf",

abstract = "

Pseudo-linear algebra is the study of common properties of linear differential and difference operators. We introduce in this paper its basic objects (pseudo-derivations, skew polynomials, and pseudo-linear operators) and describe several recent algorithms on them, which, when

applied in the differential and difference cases, yield algorithms for uncoupling and solving systems of linear differential and difference equations in closed form."

— ignore —

\bibitem[Bronstein xb]{Broxb} Bronstein, Manuel

title = "Computer Algebra Algorithms for Linear Ordinary Differential and Difference equations",
url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/ecm3.pdf",
paper = "Broxb.pdf",

abstract = "

Galois theory has now produced algorithms for solving linear ordinary differential and difference equations in closed form. In addition, recent algorithmic advances have made those algorithms effective and implementable in computer algebra systems. After introducing the relevant parts of the theory, we describe the latest algorithms for solving such equations."

— ignore —

\bibitem[Bronstein 94]{Bro94} Bronstein, Manuel

title = "An improved algorithm for factoring linear ordinary differential operators",
url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html", abstract = "

We describe an efficient algorithm for computing the associated equations appearing in the Beke-Schlesinger factorisation method for linear ordinary differential operators. This algorithm, which is based on elementary operations with sets of integers, can be easily implemented for operators of any order, produces several possible associated equations, of which only the simplest can be selected for solving, and often avoids the degenerate case, where the order of the associated equation is less than in the generic case. We conclude with some fast heuristics that can produce some factorizations while using only linear computations."

72 CHAPTER 2. THE BIBLIOGRAPHY \bibitem[Bronstein 90]{Bro90} Bronstein, Manuel title = "On Solutions of Linear Ordinary Differential Equations in their Coefficient Field", url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html", paper = "Bro90.pdf", abstract = " We describe a rational algorithm for finding the denominator of any solution of a linear ordinary differential equation in its coefficient field. As a consequence, there is now a rational algorithm for finding all such solutions when the coefficients can be built up from the rational functions by finitely many algebraic and primitive adjunctions. This also eliminates one of the computational bottlenecks in algorithms that either factor or search for Liouvillian solutions of such equations with Liouvillian coefficients." — ignore — \bibitem[Bronstein 96]{Bro96} Bronstein, Manuel "\$\sum^{IT}\$ -- A strongly-typed embeddable computer algebra library", url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html", paper = "Bro96.pdf", abstract = " We describe the new computer algebra library \$\sum^{IT}\$ and its underlying design. The development of \$\sum^{IT}\\$ is motivated by the need to provide highly efficient implementations of key algorithms for linear ordinary differential and (\$q\$)-difference equations to scientific programmers and to computer algebra users, regardless of the programming language or interactive system they use. As such, \$\sum^{IT}\$ is not a computer algebra system per se, but a library (or substrate) which is designed to be ''plugged'' with minimal efforts into different types of client applications."

— ignore —

\bibitem[Bronstein 99a]{Bro99a} Bronstein, Manuel
 title = "Solving linear ordinary differential equations over \$C(x,e^{\int{f(x)dx}})\$",
 url =
 "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
 paper = "Bro99a.pdf",
 abstract = "

We describe a new algorithm for computing the solutions in $\[F=C(x,e^{\infty})\]$ of linear ordinary differential equations with coefficients in \$F\$. Compared to the general algorithm, our algorithm avoids the computation of exponential solutions of equations with coefficients in C(x), as well as the solving of linear differential systems over C(x). Our method is effective and has been implemented."

— ignore —

\bibitem[Bronstein 00]{Bro00} Bronstein, Manuel

title = "On Solutions of Linear Ordinary Differential Equations in their Coefficient Field",
url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
paper = "Bro00.pdf",
abstract = "

We extend the notion of monomial extensions of differential fields, i.e. simple transcendental extensions in which the polynomials are closed under differentiation, to difference fields. The structure of such extensions provides an algebraic framework for solving generalized linear difference equations with coefficients in such fields. We then describe algorithms for finding the denominator of any solution of those equations in an important subclass of monomial extensions that includes transcendental indefinite sums and products. This reduces the general problem of finding the solutions of such equations in their coefficient fields to bounding their degrees. In the base case, this yields in particular a new algorithm for computing the rational solutions of \$q\$-difference equations with polynomial coefficients."

— ignore —

\bibitem[Bronstein 02]{Bro02} Bronstein, Manuel; Lafaille, S\'ebastien
 title = "Solutions of linear ordinary differential equations in terms of special functions",
 url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac2002.pdf", paper = "Bro02.pdf", abstract = "

We describe a new algorithm for computing special function solutions of the form y(x) = m(x)F((eta(x))) of second order linear ordinary differential equations, where m(x) is an arbitrary Liouvillian function, ca(x) is an arbitrary rational function, and F

satisfies a given second order linear ordinary differential equations. Our algorithm, which is base on finding an appropriate point transformation between the equation defining \$F\$ and the one to solve, is able to find all rational transformations for a large class of functions \$F\$, in particular (but not only) the \$_0F_1\$ and \$_1F_1\$ special functions of mathematical physics, such as Airy, Bessel, Kummer and Whittaker functions. It is also able to identify the values of the parameters entering those special functions, and can be generalized to equations of higher order."

— ignore —

\bibitem[Bronstein 03]{Bro03} Bronstein, Manuel; Trager, Barry M.
title = "A Reduction for Regular Differential Systems",
url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mega2003.pdf", paper = "Bro03.pdf", abstract = "

We propose a definition of regularity of a linear differential system with coefficients in a monomial extension of a differential field, as well as a global and truly rational (i.e. factorisation-free) iteration that transforms a system with regular finite singularites into an equivalent one with simple finite poles. We then apply our iteration to systems satisfied by bases of algebraic function fields, obtaining algorithms for computing the number of irreducible components and the genus of algebraic curves."

— ignore —

\bibitem[Bronstein 03a]{Bro03a} Bronstein, Manuel; Sol\'e, Patrick
 title = "Linear recurrences with polynomial coefficients",
 url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html",
paper = "Bro03a.pdf",
abstract = "

We relate sequences generated by recurrences with polynomial coefficients to interleaving and multiplexing of sequences generated by recurrences with constant coefficients. In the special case of finite fields, we show that such sequences are periodic and provide linear complexity estimates for all three constructions."

— ignore —

\bibitem[Bronstein 05]{Bro05} Bronstein, Manuel; Li, Ziming; Wu, Min title = "Picard-Vessiot Extensions for Linear Functional Systems", url =

"http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac2005.pdf", paper = "Bro05.pdf", abstract = "

Picard-Vessiot extensions for ordinary differential and difference equations are well known and are at the core of the associated Galois theories. In this paper, we construct fundamental matrices and Picard-Vessiot extensions for systems of linear partial functional equations having finite linear dimension. We then use those extensions to show that all the solutions of a factor of such a system can be completed to solutions of the original system."

— ignore —

\bibitem[Davenport 86]{Dav86} Davenport, J.H.
 title = "The Risch Differential Equation Problem",
 year = "1986",
SIAM J. COMPUT. Vol 15, No. 4
 paper = "Dav86.pdf",
 abstract = "

We propose a new algorithm, similar to Hermite's method for the integration of rational functions, for the resolution of Risch differential equations in closed form, or proving that they have no resolution. By requiring more of the presentation of our differential fields (in particular that the exponentials be weakly normalized), we can avoid the introduction of arbitrary constants which have to be solved for later.

We also define a class of fields known as exponentially reduced, and show that solutions of Risch differential equations which arise from integrating in these fields satisfy the ''natural'' degree constraints in their main variables, and we conjecture (after Risch and Norman) that this is true in all variables."

\bibitem[Singer 9]{Sing91.pdf} Singer, Michael F.

title = "Liouvillian Solutions of Linear Differential Equations with Liouvillian Coefficients" J. Symbolic Computation V11 No 3 pp251-273

year = "1991",
url = "http://www.sciencedirect.com/science/article/pii/S074771710880048X",
paper = "Sing91.pdf",
abstract = "

Let L(y)=b be a linear differential equation with coefficients in a differential field K. We discuss the problem of deciding if such an equation has a non-zero solution in K and give a decision procedure in case K is an elementary extension of the field of rational functions or is an algebraic extension of a transcendental liouvillian extension of the field of rational functions K show how one can use this result to give a procedure to find a basis for the space of solutions, liouvillian over K, of L(y)=0 where K is such a field and L(y) has coefficients in K."

— ignore —

\bibitem[Von Mohrenschildt 94]{Mohr94} {von Mohrenschildt}, Martin
 title = "Symbolic Solutions of Discontinuous Differential Equations",
 url = "http://e-collection.library.ethz.ch/eserv/eth:39463/eth-39463-01.pdf",
 paper = "Mohr94.pdf",

— ignore —

\bibitem[Von Mohrenschildt 98]{Mohr98} von Mohrenschildt, Martin
 title = "A Normal Form for Function Rings of Piecewise Functions",
J. Symbolic Computation (1998) Vol 26 pp607-619
 url = "http://www.cas.mcmaster.ca/~mohrens/JSC.pdf",
 paper = "Mohr98.pdf",

abstract = "

Computer algebra systems often have to deal with piecewise continuous functions. These are, for example, the absolute value function, signum, piecewise defined functions but also functions that are the supremum or infimum of two functions. We present a new algebraic approach to these types of problems. This paper presents a normal form for a function ring containing piecewise polynomial functions of an expression. The main result is that this normal form can be used to decide extensional equality of two piecewise functions. Also we define supremum and infimum for piecewise functions; in fact, we show that the function ring forms a lattice. Additionally, a method to solve

equalities and inequalities in this function ring is presented. Finally, we give a ''user interface'' to the algebraic representation of the piecewise functions."

— ignore —

\bibitem[Weber 06]{Webe06} Weber, Andreas

title = "Quantifier Elimination on Real Closed Fields and Differential Equations", url =

"http://cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber2006a.pdf", paper = "Webe06.pdf",

keywords = "survey",

abstract = "

This paper surveys some recent applications of quantifier elimination on real closed fields in the context of differential equations. Although polynomial vector fields give rise to solutions involving the exponential and other transcendental functions in general, many questions can be settled within the real closed field without referring to the real exponential field. The technique of quantifier elimination on real closed fields is not only of theoretical interest, but due to recent advances on the algorithmic side including algorithms for the simplification of quantifier-free formulae the method has gained practical applications, e.g. in the context of computing threshold conditions in epidemic modeling."

— ignore —

\bibitem[Ulmer 03]{Ulm03} Ulmer, Felix

title = "Liouvillian solutions of third order differential equations",

J. Symbolic COmputations 36 pp 855-889

year = "2003",

url = "http://www.sciencedirect.com/science/article/pii/S0747717103000658",
paper = "Ulm03.pdf",

abstract = "

The Kovacic algorithm and its improvements give explicit formulae for the Liouvillian solutions of second order linear differential equations. Algorithms for third order differential equations also exist, but the tools they use are more sophisticated and the computations more involved. In this paper we refine parts of the algorithm to find Liouvillian solutions of third order equations. We show that, except for four finite groups and a reduction to the second order case, it is possible to give a formula in the imprimitve

case. We also give necessary conditions and several simplifications for the computation of the minimal polynomial for the remaining finite set of finite groups (or any known finite group) by extracting ramification information from the character table. Several examples have been constructed, illustrating the possibilities and limitations."

2.22 Expression Simplification

— ignore —

\bibitem[Carette 04]{Car04} Carette, Jacques
 title = "Understanding Expression Simplification",
 url = "http://www.cas.mcmaster.ca/~carette/publications/simplification.pdf",
 paper = "Car04.pdf",
 abstract = "
 We give the first formal definition of the concept of {\sl
 simplification} for general expressions in the context of Computer
 Algebra Systems. The main mathematical tool is an adaptation of the
 theory of Minimum Description Length, which is closely related to
 various theories of complexity, such as Kolmogorov Complexity and
 Algorithmic Information Theory. In particular, we show how this theory
 can justify the use of various ''magic constants'' for deciding
 between some equivalent representations of an expression, as found in
 implementations of simplification routines."

2.23 Integration

- axiom.bib -

```
@TechReport{Kalt84b,
   author = "Kaltofen, E.",
   title = "The Algebraic Theory of Integration",
   institution = "RPI",
   address = "Dept. Comput. Sci., Troy, New York",
   year = "1984",
   url =
        "http://www.math.ncsu.edu/~kaltofen/bibliography/84/Ka84_integration.pdf",
   paper = "Kalt84b.pdf",
```

}

— ignore —

\bibitem[Adamchik xx]{Adamxx} Adamchik, Victor
 title = "Definite Integration",
 url = "http://www.cs.cmu.edu/~adamchik/articles/integr/mj.pdf",
 paper = "Adamxx.pdf",

— ignore —

\bibitem[Adamchik 97]{Adam97} Adamchik, Victor
 title = "A Class of Logarithmic Integrals",
 url = "http://www.cs.cmu.edu/~adamchik/articles/issac/issac97.pdf",
 paper = "Adam97.pdf",
 abstract = "

A class of definite integrals involving cyclotomic polynomials and nested logarithms is considered. The results are given in terms of derivatives of the Hurwitz Zeta function. Some special cases for which such derivatives can be expressed in closed form are also considered."

— ignore —

\bibitem[Avgoustis 77]{Avgo77} Avgoustis, Ioannis Dimitrios
title =

"Definite Integration using the Generalized Hypergeometric Functions", url = "http://dspace.mit.edu/handle/1721.1/16269", paper = "Avgo77.pdf", abstract = "

A design for the definite integration of approximately fifty Special Functions is described. The Generalized Hypergeometric Functions are utilized as a basis for the representation of the members of the above set of Special Functions. Only a relatively small number of formulas that generally involve Generalized Hypergeometric Functions are utilized for the integration stage. A last and crucial stage is required for the integration process: the reduction of the Generalized Hypergeometric Function to Elementary and/or Special Functions.

The result of an early implementation which involves Laplace transforms are given and some actual examples with their corresponding timing are provided."

- ignore -

\bibitem[Baddoura 89]{Bad89} Baddoura, Jamil

title = "A Dilogarithmic Extension of Liouville's Theorem on Integration in Finite Terms",

url = "http://www.dtic.mil/dtic/tr/fulltext/u2/a206681.pdf",
paper = "Bad89.pdf",
abstract = "

The result obtained generalizes Liouville's Theorem by allowing, in addition to the elementary functions, dilogarithms to appear in the integral of an elementary function. The basic conclusion is that an associated function to the dilogarithm, if dilogarithms appear in the integral, appears linearly, with logarithms appearing in a non-linear way."

— ignore —

\bibitem[Baddoura 94]{Bad94} Baddoura, Mohamed Jamil

url = "http://dspace.mit.edu/bitstream/handle/1721.1/26864/30757785.pdf",
paper = "Bad94.pdf",
abstract = "

In this thesis, we report on a new theorem that generalizes Liouville's theorem on integration in finite terms. The new theorem allows dilogarithms to occur in the integral in addition to elementary functions. The proof is base on two identities for the dilogarithm, that characterize all the possible algebraic relations among dilogarithms of functions that are built up from the rational functions by taking transcendental exponentials, dilogarithms, and logarithms."

\bibitem[Baddoura 10]{Bad10} Baddoura, Jamil
 title = "A Note on Symbolic Integration with Polylogarithms",
 year = "2011",
J. Math Vol 8 pp229-241 (2011)
 paper = "Bad10.pdf",
 abstract = "
 We generalize partially Liouville's theorem on integration in finite
 terms to allow polylogarithms of any order to occur in the integral in
 addition to elementary functions. The result is a partial
 generalization of a theorem proved by the author for the

addition to elementary functions. The result is a partial generalization of a theorem proved by the author for the dilogarithm. It is also a partial proof of a conjecture postulated by the author in 1994. The basic conclusion is that an associated function to the nth polylogarithm appears linearly with logarithms appearing possibly in a polynomial way with non-constant coefficients."

— ignore —

\bibitem[Bajpai 70]{Bajp70} Bajpai, S.D.

title = "A contour integral involving legendre polynomial and Meijer's G-function",
url = "http://link.springer.com/article/10.1007/BF03049565",
paper = "Bajp70.pdf",
abstract = "

In this paper a countour integral involving Legendre polynomial and Meijer's G-function is evaluated. the integral is of general character and it is a generalization of results recently given by Meijer, MacRobert and others. An integral involving regular radial Coulomb wave function is also obtained as a particular case."

— ignore —

\bibitem[Bronstein 89]{Bro89a} Bronstein, M.

title = "An Algorithm for the Integration of Elementary Functions", Lecture Notes in Computer Science Vol 378 pp491-497 $\,$

year = "1989",
paper = "Bro89a.pdf",

abstract = "

Trager (1984) recently gave a new algorithm for the indefinite integration of algebraic functions. His approach was ''rational'' in the sense that the only algebraic extension computed in the smallest one necessary to express the answer. We outline a generalization of this approach that allows us to integrate mixed elementary functions. Using only rational techniques, we are able to normalize

the integrand, and to check a necessary condition for elementary integrability."

— ignore —

\bibitem[Bronstein 90a]{Bro90a} Bronstein, Manuel
 title = "Integration of Elementary Functions",
J. Symbolic Computation 9, pp117-173
 year = "1990",
 paper = "Bro90a.pdf",
 abstract = "

We extend a recent algorithm of Trager to a decision procedure for the indefinite integration of elementary functions. We can express the integral as an elementary function or prove that it is not elementary. We show that if the problem of integration in finite terms is solvable on a given elementary function field \$k\$, then it is solvable in any algebraic extension of \$k(\theta)\$, where \$\theta\$ is a logarithm or exponential of an element of \$k\$. Our proof considers an element of such an extension field to be an algebraic function of one variable over \$k\$.

In his algorithm for the integration of algebraic functions, Trager describes a Hermite-type reduction to reduce the problem to an integrand with only simple finite poles on the associated Riemann surface. We generalize that technique to curves over liouvillian ground fields, and use it to simplify our integrands. Once the multipe finite poles have been removed, we use the Puiseux expansions of the integrand at infinity and a generalization of the residues to compute the integral. We also generalize a result of Rothstein that gives us a necessary condition for elementary integrability, and provide examples of its use."

— axiom.bib —

```
@article{Bron90c,
   author = "Bronstein, Manuel",
   title = "On the integration of elementary functions",
   journal = "Journal of Symbolic Computation",
   volume = "9",
   number = "2",
   pages = "117-173",
   year = "1990",
```

```
month = "February"
}
```

— ignore —

\bibitem[Bronstein 93]{REF-BS93} Bronstein, Manuel; Salvy, Bruno
 title = "Full partial fraction decomposition of rational functions",
In Bronstein [Bro93] pp157-160 ISBN 0-89791-604-2 LCCN QA76.95 I59 1993
 url = "http://www.acm.org/pubs/citations/proceedings/issac/164081/",

— ignore —

\bibitem[Bronstein 90]{Bro90b} Bronstein, Manuel
 title = "A Unification of Liouvillian Extensions",
 paper = "Bro90b.pdf",
 abstract = "

We generalize Liouville's theory of elementary functions to a larger class of differential extensions. Elementary, Liouvillian and trigonometric extensions are all special cases of our extensions. In the transcendental case, we show how the rational techniques of integration theory can be applied to our extensions, and we give a unified presentation which does not require separate cases for different monomials."

— axiom.bib —

```
@book{Bron97,
   author = "Bronstein, Manuel",
   title = "Symbolic Integration I--Transcendental Functions",
   publisher = "Springer, Heidelberg",
   year = "1997",
   isbn = "3-540-21493-3",
   url = "http://evil-wire.org/arrrXiv/Mathematics/Bronstein,_Symbolic_Integration_I,1997.pdf",
   paper = "Bron97.pdf",
}
```

@article{Bron07,

author = "Bronstein, Manuel",

title = "Structure theorems for parallel integration",

journal = "Journal of Symbolic Computation",

— ignore — \bibitem[Bronstein 05a]{Bro05a} Bronstein, Manuel title = "The Poor Man's Integrator, a parallel integration heuristic", url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/pmint/pmint.txt", url2 = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/pmint/examples", paper = "Bro05a.txt", — axiom.bib — @article{Bron06, author = "Bronstein, M.", title = "Parallel integration", journal = "Programming and Computer Software", year = "2006",issn = "0361-7688",volume = "32",number = "1", doi = "10.1134/S0361768806010075", url = "http://dx.doi.org/10.1134/S0361768806010075", publisher = "Nauka/Interperiodica", pages = "59-60", paper = "Bron06.pdf", abstract = " Parallel integration is an alternative method for symbolic integration. While also based on Liouville's theorem, it handles all the generators of the differential field containing the integrand 'in parallel'', i.e. all at once rather than considering only the topmost one in a recursive fasion. Although it still contains heuristic aspects, its ease of implementation, speed, high rate of success, and ability to integrate functions that cannot be handled by the Risch algorithm make it an attractive alternative." } — axiom.bib —

```
volume = "42",
number = "7",
pages = "757-769",
year = "2007",
month = "July",
paper = "Bron07.pdf",
abstract = "

We introduce structure theorems that refine Liouville's Theorem on
integration in closed form for general derivations on multivariate
rational function fields. By predicting the arguments of the new
logarithms that an appear in integrals, as well as the denominator of
the rational part, those theorems provide theoretical backing for the
Risch-Norman integration method. They also generalize its applicability
to non-monomial extensions, for example the Lambert W function."
}
```

— ignore —

```
\bibitem[Charlwood 07]{Charl07} Charlwood, Kevin
  title = "Integration on Computer Algebra Systems",
The Electronic J of Math. and Tech. Vol 2, No 3, ISSN 1933-2823
  url = "http://12000.org/my_notes/ten_hard_integrals/paper.pdf",
  paper = "Charl07.pdf",
  abstract = "
```

In this article, we consider ten indefinite integrals and the ability of three computer algebra systems (CAS) to evaluate them in closed-form, appealing only to the class of real, elementary functions. Although these systems have been widely available for many years and have undergone major enhancements in new versions, it is interesting to note that there are still indefinite integrals that escape the capacity of these systems to provide antiderivatves. When this occurs, we consider what a user may do to find a solution with the aid of a CAS."

```
\bibitem[Charlwood 08]{Charl08} Charlwood, Kevin
  title = "Symbolic Integration Problems",
  url = "http://www.apmaths.uwo.ca/~arich/IndependentTestResults/CharlwoodIntegrationProblems.pdf",
  paper = "Charl08.pdf",
  abstract = "
    A list of the 50 example integration problems from Kevin Charlwood's 2008
```

article ''Integration on Computer Algebra Systems''. Each integral along with its optimal antiderivative (that is, the best antiderivative found so far) is shown."

— ignore —

\bibitem[Cherry 84]{Che84} Cherry, G.W.

title = "Integration in Finite Terms with Special Functions: The Error Function", J. Symbolic Computation (1985) Vol 1 pp283-302

paper = "Che84.pdf",

abstract = "

A decision procedure for integrating a class of transcendental elementary functions in terms of elementary functions and error functions is described. The procedure consists of three mutually exclusive cases. In the first two cases a generalised procedure for completing squares is used to limit the error functions which can appear in the integral of a finite number. This reduces the problem to the solution of a differential equation and we use a result of Risch (1969) to solve it. The third case can be reduced to the determination of what we have termed \$\sum\$-decompositions. The result presented here is the key procuedure to a more general algorithm which is described fully in Cherry (1983)."

— ignore —

\bibitem[Cherry 86]{Che86} Cherry, G.W.

title = "Integration in Finite Terms with Special Functions: The Logarithmic Integral", SIAM J. Comput. Vol 15 pp1-21 February 1986

— ignore —

\bibitem[Cherry 89]{Che89} Cherry, G.W.

title = "An Analysis of the Rational Exponential Integral",

SIAM J. Computing Vol 18 pp 893-905 (1989)

paper = "Che89.pdf",

abstract = "

In this paper an algorithm is presented for integrating expressions of

the form \$\int{ge^f^dx}\$, where \$f\$ and \$g\$ are rational functions of \$x\$, in terms of a class of special functions called the special incomplete \$\Gamma\$ functions. This class of special functions includes the exponential integral, the error functions, the sine and cosing integrals, and the Fresnel integrals. The algorithm presented here is an improvement over those published previously for integrating with special functions in the following ways: (i) This algorithm combines all the above special functions into one algorithm, whereas previously they were treated separately, (ii) Previous algorithms require that the underlying field of constants be algebraically closed. This algorithm, however, works over any field of characteristic zero in which the basic field operations can be carried out. (iii) This algorithm does not rely on Risch's solution of the differential equation \$y^\prime + fy = g\$. Instead, a more direct method of undetermined coefficients is used."

— ignore —

\bibitem[Churchill 06]{Chur06} Churchill, R.C.

title = "Liouville's Theorem on Integration Terms of Elementary Functions",
url = "http://www.sci.ccny.cuny.edu/~ksda/PostedPapers/liouv06.pdf",
paper = "Chur06.pdf",
abstract = "

This talk should be regarded as an elementary introduction to differential algebra. It culminates in a purely algebraic proof, due to M. Rosenlicht, of an 1835 theorem of Liouville on the existence of ''elementary'' integrals of ''elementary'' functions. The precise meaning of elementary will be specified. As an application of that theorem we prove that the indefinite integral $\pi^2 \$ cannot be expressed in terms of elementary functions.

\begin{itemize}

 $\$ item Preliminaries on Meromorphic Functions

\item Basic (Ordinary) Differential Algebra

 $\$ Differential Ring Extensions with No New Constants

\item Extending Derivations

\item Integration in Finite Terms

\end{itemize}"

— ignore —

\bibitem[Davenport 79b]{Dav79b} Davenport, James Harold
 title = "On the Integration of Algebraic Functions",

Springer-Verlag Lecture Notes in Computer Science 102 ISBN 0-387-10290-6

- ignore -

\bibitem[Davenport 79c]{Dav79c} Davenport, J. H.
 title = "Algorithms for the Integration of Algebraic Functions",
Lecture Notes in Computer Science V 72 pp415-425 (1979)
 paper = "Dav79c.pdf",
 abstract = "

The problem of finding elementary integrals of algebraic functions has long been recognized as difficult, and has sometimes been thought insoluble. Risch stated a theorem characterising the integrands with elementary integrals, and we can use the language of algebraic geometry and the techniques of Davenport to yield an algorithm that will always produce the integral if it exists. We explain the difficulty in the way of extending this algorithm, and outline some ways of solving it. Using work of Manin we are able to solve the problem in all cases where the algebraic expressions depend on a parameter as well as on the variable of integration."

— ignore —

\bibitem[Davenport 82a]{Dav82a} Davenport, J.H.
 title = "The Parallel Risch Algorithm (I)"
 paper = "Dav82a.pdf",
 abstract = "

In this paper we review the so-called ''parallel Risch'' algorithm for the integration of transcendental functions, and explain what the problems with it are. We prove a positive result in the case of logarithmic integrands."

— ignore —

\bibitem[Davenport 82]{Dav82} Davenport, J.H.
 title = "On the Parallel Risch Algorithm (III): Use of Tangents",
SIGSAM V16 no. 3 pp3-6 August 1982

```
— ignore —
\bibitem[Davenport 03]{Dav03} Davenport, James H.
  title = "The Difficulties of Definite Integration",
  url = "http://www.researchgate.net/publication/247837653_The_Diculties_of_Definite_Integration/file/72e7e
 paper = "Dav03.pdf",
  abstract = "
    Indefinite integration is the inverse operation to differentiation,
    and, before we can understand what we mean by indefinite integration,
    we need to understand what we mean by differentiation."
            — ignore —
\bibitem[Fateman 02]{Fat02} Fateman, Richard
  title = "Symbolic Integration",
  url = "http://inst.eecs.berkeley.edu/~cs282/sp02/lects/14.pdf",
 paper = "Fat02.pdf",
            — axiom.bib —
@inproceedings{Gedd89,
  author = "Geddes, K. O. and Stefanus, L. Y.",
  title = "On the Risch-norman Integration Method and Its Implementation
          in MAPLE",
  booktitle = "Proc. of the ACM-SIGSAM 1989 Int. Symp. on Symbolic and
              Algebraic Computation",
  series = "ISSAC '89",
  year = "1989",
  isbn = "0-89791-325-6",
  location = "Portland, Oregon, USA",
  pages = "212--217",
  numpages = "6",
  url = "http://doi.acm.org/10.1145/74540.74567",
  doi = "10.1145/74540.74567",
  acmid = "74567",
 publisher = "ACM",
  address = "New York, NY, USA",
 paper = "Gedd89.pdf",
  abstract = "
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}

Unlike the Recursive Risch Algorithm for the integration of transcendental elementary functions, the Risch-Norman Method processes the tower of field extensions directly in one step. In addition to logarithmic and exponential field extensions, this method can handle extentions in terms of tangents. Consequently, it allows trigonometric functions to be treated without converting them to complex exponential form. We review this method and describe its implementation in MAPLE. A heuristic enhancement to this method is also presented."

- ignore -

\bibitem[Geddes 92a]{GCL92a} Geddes, K.O.; Czapor, S.R.; Labahn, G.
 title = "The Risch Integration Algorithm",
Algorithms for Computer Algebra, Ch 12 pp511-573 (1992)
 paper = "GCL92a.pdf",

— ignore —

\bibitem[Hardy 1916]{Hard16} Hardy, G.H.
 title = "The Integration of Functions of a Single Variable",
Cambridge University Press, Cambridge, 1916
% REF:00002

— ignore —

\bibitem[Harrington 78]{Harr87} Harrington, S.J.
 title = "A new symbolic integration system in reduce",
 url = "http://comjnl.oxfordjournals.or/content/22/2/127.full.pdf",
 paper = "Harr87.pdf",
 abstract = "

A new integration system, employing both algorithmic and pattern match integration schemes is presented. The organization of the system differs from that of earlier programs in its emphasis on the algorithmic approach to integration, its modularity and its ease of revision. The new Norman-Rish algorithm and its implementation at the University of Cambridge are employed, supplemented by a powerful

collection of simplification and transformation rules. The facility for user defined integrals and functions is also included. The program is both fast and powerful, and can be easily modified to incorporate anticipated developments in symbolic integration."

— axiom.bib —

```
@misc{Herm1872,
  author = "Hermite, E.",
  title = "Sur l'int\'{e}gration des fractions rationelles",
  journal = "Nouvelles Annales de Math\'{e}matiques",
  volume = "11",
  pages = "145-148",
  year = "1872"
}
```

— ignore —

\bibitem[Horowitz 71]{Horo71} Horowitz, Ellis
title = "Algorithms for Partial Fraction Decomposition and Rational Function Integration",
SYMSAC '71 Proc. ACM Symp. on Symbolic and Algebraic Manipulation (1971)
pp441-457

```
paper = "Horo71.pdf",
ref = "00018",
abstract = "
```

Algorithms for symbolic partial fraction decomposition and indefinite integration of rational functions are described. Two types of partial fraction decomposition are investigated, square-free and complete square-free. A method is derived, based on the solution of a linear system, which produces the square-free decomposition of any rational function, say A/B. The computing time is show to be $0(n^4(\ln nf)^2)$ where ${\rm method}(A) < {\rm mhdeg}(B) = n$ and f is a number which is closely related to the size of the coefficients which occur in A and B. The complete square-free partical fraction decomposition can then be directly obtained and it is shown that the computing time for this process is also bounded by $0(n^4(\ln nf)^2)$."

\bibitem[Jeffrey 97]{Jeff97} Jeffrey, D.J.; Rich, A.D.
 title = "Recursive integration of piecewise-continuous functions",
 url = "http://www.cybertester.com/data/recint.pdf",
 paper = "Jeff97.pdf",
 abstract = "

An algorithm is given for the integration of a class of piecewise-continuous functions. The integration is with respect to a real variable, because the functions considered do not in general allow integration in the complex plane to be defined. The class of integrands includes commonly occurring waveforms, such as square waves, triangular waves, and the floor function; it also includes the signum function. The algorithm can be implemented recursively, and it has the property of ensuring that integrals are continuous on domains of maximum extent."

— ignore —

\bibitem[Jeffrey 99]{Jeff99} Jeffrey, D.J.; Labahn, G.; Mohrenschildt, M.v.; Rich, A.D.

title = "Integration of the signum, piecewise and related functions",
url = "http://cs.uwaterloo.ca/~glabahn/Papers/issac99-2.pdf",
paper = "Jeff99.pdf",
abstract = "

When a computer algebra system has an assumption facility, it is possible to distinguish between integration problems with respect to a real variable, and those with respect to a complex variable. Here, a class of integration problems is defined in which the integrand consists of compositions of continuous functions and signum functions, and integration is with respect to a real variable. Algorithms are given for evaluating such integrals."

— ignore —

\bibitem[Kiymaz 04]{Kiym04} Kiymaz, Onur; Mirasyedioglu, Seref
 title = "A new symbolic computation for formal integration with exact power series",
 paper = "Kiym04.pdf",
 abstract = "

This paper describes a new symbolic algorithm for formal integration of a class of functions in the context of exact power series by using generalized hypergeometric series and computer algebraic technique."

— ignore —

\bibitem[Knowles 93]{Know93} Knowles, P.

title = "Integration of a class of transcendental liouvillian functions with error-functions i", Journal of Symbolic Computation Vol 13 pp525-543 (1993)

— ignore —

\bibitem[Knowles 95]{Know95} Knowles, P.

title = "Integration of a class of transcendental liouvillian functions with error-functions ii", Journal of Symbolic Computation Vol 16 pp227-241 (1995)

— axiom.bib —

```
@article{Krag09,
```

```
author = "Kragler, R.",
  title = "On Mathematica Program for Poor Man's Integrator Algorithm",
  journal = "Programming and Computer Software",
  volume = "35",
  number = "2",
  pages = "63-78",
 year = "2009",
  issn = "0361-7688",
 paper = "Krag09.pdf",
  abstract = "
    In this paper by means of computer experiment we study advantages and
    disadvantages of the heuristical method of ''parallel integrator''. For
    this purpose we describe and use implementation of the method in
    Mathematica. In some cases we compare this implementation with the original
    one in Maple."
}
```

— ignore —

\bibitem[Lang 93]{Lang93} Lang, S.

```
title = "Algebra",
Addison-Wesly, New York, 3rd edition 1993
```

- ignore -

\bibitem[Leerawat 02]{Leer02} Leerawat, Utsanee; Laohakosol, Vichian
 title = "A Generalization of Liouville's Theorem on Integration in Finite Terms",
 url = "http://www.mathnet.or.kr/mathnet/kms_tex/113666.pdf",
 paper = "Leer02.pdf",
 abstract = "

A generalization of Liouville's theorem on integration in finite terms, by enlarging the class of fields to an extension called Ei-Gamma extension is established. This extension includes the \$\mathcal{E}\mathcal{L}\$-elementary extensions of Singer, Saunders and Caviness and contains the Gamma function."

— ignore —

\bibitem[Leslie 09]{Lesl09} Leslie, Martin
 title = "Why you can't integrate exp(\$x^2\$)",
 url = "http://math.arizona.edu/~mleslie/files/integrationtalk.pdf",
 paper = "Lesl09.pdf",

— ignore —

\bibitem[Lichtblau 11]{Lich11} Lichtblau, Daniel
 title = "Symbolic definite (and indefinite) integration: methods and open issues",

ACM Comm. in Computer Algebra Issue 175, Vol 45, No.1 (2011)
url = "http://www.sigsam.org/bulletin/articles/175/issue175.pdf",
paper = "Lich11.pdf",
abstract = "

The computation of definite integrals presents one with a variety of choices. There are various methods such as Newton-Leibniz or Slater's convolution method. There are questions such as whether to split or merge sums, how to search for singularities on the path of integration, when to issue conditional results, how to assess (possibly conditional) convergence, and more. These various

— ignore —

considerations moreover interact with one another in a multitude of ways. Herein we discuss these various issues and illustrate with examples."

```
— axiom.bib —
@article{Liou1833a,
  author = "Liouville, Joseph",
  title = "Premier m\'{e}moire sur la d\'{e}termination des int\'{e}grales
           dont la valeur est alg\'{e}brique",
  journal = "Journal de l'Ecole Polytechnique",
  volume = "14",
 pages = "124-128",
 year = "1833"
            — axiom.bib —
@article{Liou1833b,
  author = "Liouville, Joseph",
 title = "Second m\'{e}moire sur la d\'{e}termination des int\'{e}grales
          dont la valeur est alg\'{e}brique",
  journal = "Journal de l'Ecole Polytechnique",
  volume = "14",
 pages = "149-193",
 year = "1833"
           — ignore —
\bibitem[Liouville 1833c]{Lio1833c} Liouville, Joseph
 title = "Note sur la determination des int\'egrales dont la valeur est alg\'ebrique",
Journal f\"ur die Reine und Angewandte Mathematik,
Vol 10 pp 247-259, (1833)
```

\bibitem[Liouville 1833d] Liouville, Joseph
 title = "Sur la determination des int\'egrales dont la valeur est alg\'ebrique",
{\sl Journal de l'Ecole Polytechnique}, 14:124-193, 1833

— ignore —

\bibitem[Liouville 1835]{Lio1835} Liouville, Joseph
 title = "M\'emoire sur l'int\'gration d'une classe de fonctions transcendentes",
Journal f\"ur die Reine und Angewandte Mathematik,
Vol 13(2) pp 93-118, (1835)

— ignore —

\bibitem[Marc 94]{Marc94} Marchisotto, Elena Anne; Zakeri, Gholem-All
 title = "An Invitation to Integration in Finite Terms",
College Mathematics Journal Vol 25 No 4 (1994) pp295-308
 url = "http://www.rangevoting.org/MarchisottoZint.pdf",
 paper = "Marc94.pdf",

— ignore —

\bibitem[Marik 91]{Mari91} Marik, Jan
 title = "A note on integration of rational functions",
 url = "http://dml.cz/bitstream/handle/10338.dmlcz/126024/MathBohem_116-1991-4_9.pdf",
 paper = "Mari91.pdf",
 abstract = "
 Let \$P\$ and \$Q\$ be polynomials in one variable with complex coefficients
 and let \$n\$ be a natural number. Suppose that \$Q\$ is not constant and

Let P\$ and Q\$ be polynomials in one variable with complex coefficients and let n\$ be a natural number. Suppose that Q\$ is not constant and has only simple roots. Then there is a rational function α with $\alpha P/Q^{n+1}$ \$ if and only if the Wronskian of the functions Q^p^{n+1} \$ if and only if the Wronskian of the functions Q^p^{n+1} \$ is divisible by Q\$."

\bibitem[Moses 76]{Mos76} Moses, Joel
 title = "An introduction to the Risch Integration Algorithm",
ACM Proc. 1976 annual conference pp425-428
 paper = "Mos76.pdf",
 ref = "00048",
 abstract = "

Risch's decision procedure for determining the integrability in closed form of the elementary functions of the calculus is presented via examples. The exponential and logarithmic cases of the algorithsm had been implemented for the MACSYMA system several years ago. The implementation of the algebraic case of the algorithm is the subject of current research."

— ignore —

\bibitem[Moses 71a]{Mos71a} Moses, Joel
 title = "Symbolic Integration: The Stormy Decade",
CACM Aug 1971 Vol 14 No 8 pp548-560
 url = "http://www-inst.eecs.berkeley.edu/~cs282/sp02/readings/moses-int.pdf",
 paper = "Mos71a.pdf",
 ref = "00017",
 abstract = "

Three approaches to symbolic integration in the 1960's are described. The first, from artificial intelligence, led to Slagle's SAINT and to a large degree to Moses' SIN. The second, from algebraic manipulation, led to Monove's implementation and to Horowitz' and Tobey's reexamination of the Hermite algorithm for integrating rational functions. The third, from mathematics, led to Richardson's proof of the unsolvability of the problem for a class of functions and for Risch's decision procedure for the elementary functions. Generalizations of Risch's algorithm to a class of special functions and programs for solving differential equations and for finding the definite integral are also described."

— ignore —

\bibitem[Norman 79]{Nor79} Norman, A.C.; Davenport, J.H.
title = "Symbolic Integration -- The Dust Settles?",
paper = "Nor79.pdf",
abstract = "

By the end of the 1960s it had been shown that a computer could find indefinite integrals with a competence exceeding that of typical ${\cal C}$

undergraduates. This practical advance was backed up by algorithmic interpretations of a number of clasical results on integration, and by some significant mathematical extensions to these same results. At that time it would have been possible to claim that all the major barriers in the way of a complete system for automated analysis had been breached. In this paper we survey the work that has grown out of the above-mentioned early results, showing where the development has been smooth and where it has spurred work in seemingly unrelated fields."

— ignore —

\bibitem[Ostrowski 46]{Ost46} Ostrowski, A.
 title = "Sur l'int\'egrabilit\'e \'el\'ementaire de quelques classes d'expressions",
Comm. Math. Helv., Vol 18 pp 283-308, (1946)
% REF:00008

— ignore —

\bibitem[Raab 12]{Raab12} Raab, Clemens G.
 title = "Definite Integration in Differential Fields",
 url = "http://www.risc.jku.at/publications/download/risc_4583/PhD_CGR.pdf",
 paper = "Raab12.pdf",
 abstract = "

The general goal of this thesis is to investigate and develop computer algebra tools for the simplification resp. evaluation of definite integrals. One way of finding the value of a def- inite integral is via the evaluation of an antiderivative of the integrand. In the nineteenth century Joseph Liouville was among the first who analyzed the structure of elementary antiderivatives of elementary functions systematically. In the early twentieth century the algebraic structure of differential fields was introduced for modeling the differential properties of functions. Using this framework Robert H. Risch published a complete algorithm for transcendental elementary integrands in 1969. Since then this result has been extended to certain other classes of integrands as well by Michael F. Singer, Manuel Bronstein, and several others. On the other hand, if no antiderivative of suitable form is available, then linear relations that are satisfied by the parameter integral of interest may be found based on the principle of parametric integration (often called differentiating under the integral sign or creative telescoping).

The main result of this thesis extends the results mentioned above to

a complete algo- rithm for parametric elementary integration for a certain class of integrands covering a majority of the special functions appearing in practice such as orthogonal polynomials, polylogarithms, Bessel functions, etc. A general framework is provided to model those functions in terms of suitable differential fields. If the integrand is Liouvillian, then the present algorithm considerably improves the efficiency of the corresponding algorithm given by Singer et al. in 1985. Additionally, a generalization of Czichowskis algorithm for computing the logarithmic part of the integral is presented. Moreover, also partial generalizations to include other types of integrands are treated.

As subproblems of the integration algorithm one also has to find solutions of linear or- dinary differential equations of a certain type. Some contributions are also made to solve those problems in our setting, where the results directly dealing with systems of differential equations have been joint work with Moulay A. Barkatou.

For the case of Liouvillian integrands we implemented the algorithm in form of our Mathematica package Integrator. Parts of the implementation also deal with more general functions. Our procedures can be applied to a significant amount of the entries in integral tables, both indefinite and definite integrals. In addition, our procedures have been successfully applied to interesting examples of integrals that do not appear in these tables or for which current standard computer algebra systems like Mathematica or Maple do not succeed. We also give examples of how parameter integrals coming from the work of other researchers can be solved with the software, e.g., an integral arising in analyzing the entropy of certain processes."

— ignore —

\bibitem[Raab 13]{Raab13} Raab, Clemens G.
title = "Generalization of Risch's Algorithm to Special Functions",

url = "http://arxiv.org/pdf/1305.1481",

paper = "Raab13.pdf",

abstract = "

Symbolic integration deals with the evaluation of integrals in closed form. We present an overview of Risch's algorithm including recent developments. The algorithms discussed are suited for both indefinite and definite integration. They can also be used to compute linear relations among integrals and to find identities for special functions given by parameter integrals. The aim of this presentation is twofold: to introduce the reader to some basic idea of differential algebra in the context of integration and to raise awareness in the physics community of computer algebra algorithms for indefinite and definite

integration."

— ignore —

\bibitem[Raab xx]{Raabxx} Raab, Clemens G.

title = "Integration in finite terms for Liouvillian functions",
url = "http://www.mmrc.iss.ac.cn/~dart4/posters/Raab.pdf",
paper = "Raabxx.pdf",
abstract = "

Computing integrals is a common task in many areas of science, antiderivatives are one way to accomplish this. The problem of integration in finite terms can be states as follows. Given a differential field (F,D) and $f \in \mathbb{R}$, compute g in some elementary extension of (F,D) such that Dg = f if such a g exists.

This problem has been solved for various classes of fields \$F\$. For rational functions $(C(x), \frac{d}{dx})$ \$ such a \$g\$ always exists and algorithms to compute it are known already for a long time. In 1969 Risch published an algorithm that solves this problem when (F,D)\$ is a transcendental elementary extension of $(C(x),\frac{d}{dx})$ \$. Later this has been extended towards integrands being Liouvillian functions by Singer et. al. via the use of regular log-explicit extensions of $(C(x),\frac{d}{dx})$ \$. Our algorithm extends this to handling transcendental Liouvillian extensions (F,D)\$ of (C,0)\$ directly without the need to embed them into log-explicit extensions. For example, this means that

 $\[\int (x-x)x^{z-1}e^{-x}dx \] = x^ze^{-x} \]$ can be computed without including $\log(x)$ in the differential field."

— ignore —

\bibitem[Rich 09]{Rich09} Rich, A.D.; Jeffrey, D.J.

title = "A Knowledge Repository for Indefinite Integration Based on Transformation Rules",
url = "http://www.apmaths.uwo.ca/~arich/A%2520Rule-based%2520Knowedge%2520Repository.pdf",
paper = "Rich09.pdf",
abstract = "

Taking the specific problem domain of indefinite integration, we describe the on-going development of a repository of mathematical knowledge based on transformation rules. It is important that the repository be not confused with a look-up table. The database of transformation rules is at present encoded in Mathematica, but this is

only one convenient form of the repository, and it could be readily translated into other formats. The principles upon which the set of rules is compiled is described. One important principle is minimality. The benefits of the approach are illustrated with examples, and with the results of comparisons with other approaches."

```
— axiom.bib —
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```
@techreport{Risc68,
  author = "Risch, Robert",
  title = "On the integration of elementary functions which are built up
          using algebraic operations",
  type = "Research Report",
 number = "SP-2801/002/00",
  institution = "System Development Corporation, Santa Monica, CA, USA",
 year = "1968"
            — axiom.bib —
@techreport{Risc69a,
  author = "Risch, Robert",
  title = "Further results on elementary functions",
  type = "Research Report",
  number = "RC-2042",
  institution = "IBM Research, Yorktown Heights, NY, USA",
 year = "1969"
}
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@article{Risc69b,
   author = "Risch, Robert",
   title = "The problem of integration in finite terms",
   journal = "Transactions of the American Mathematical Society",
   volume = "139",
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```
year = "1969",
pages = "167-189",
paper = "Ris69b.pdf",
abstract = "This paper deals with the problem of telling whether a
   given elementary function, in the sense of analysis, has an elementary
   indefinite integral."
}
```

— axiom.bib —

```
@article{Risc70,
   author = "Risch, Robert",
   title = "The Solution of the Problem of Integration in Finite Terms",
   journal = "Bull. AMS",
   year = "1970",
   issn = "0002-9904",
   volume = "76",
   number = "3",
   pages = "605-609",
   paper = "Risc70.pdf",
   abstract = "
The problem of integration in finite terms asks for an algorithm for the problem.
```

The problem of integration in finite terms asks for an algorithm for deciding whether an elementary function has an elementary indefinite integral and for finding the integral if it does. "Elementary" is used here to denote those functions build up from the rational functions using only exponentiation, logarithms, trigonometric, inverse trigonometric and algebraic operations. This vaguely worded question has several precise, but inequivalent formulations. The writer has devised an algorithm which solves the classical problem of Liouville. A complete account is planned for a future publication. The present note is intended to indiciate some of the ideas and techniques involved."

```
@article{Risc79,
   author = "Risch, Robert",
   title = "Algebraic properties of the elementary functions of analysis",
   journal = "American Journal of Mathematics",
   volume = "101",
   pages = "743-759",
```

```
year = "1979"
}
```

— ignore —

\bibitem[Ritt 48]{Ritt48} Ritt, J.F.
 title = "Integration in Finite Terms",
Columbia University Press, New York 1948
% REF:00046

— ignore —

\bibitem[Rosenlicht 68]{Ro68} Rosenlicht, Maxwell
 title = "Liouville's Theorem on Functions with Elementary Integrals",
Pacific Journal of Mathematics Vol 24 No 1 (1968)
 url = "http://msp.org/pjm/1968/24-1/pjm-v24-n1-p16-p.pdf",
 paper = "Ro68.pdf",
 ref = "00047",
 abstract = "

Defining a function with one variable to be elemetary if it has an explicit representation in terms of a finite number of algebraic operations, logarithms, and exponentials. Liouville's theorem in its simplest case says that if an algebraic function has an elementary integral then the latter is itself an algebraic function plus a sum of constant multiples of logarithms of algebraic functions. Ostrowski has generalized Liouville's results to wider classes of meromorphic functions on regions of the complex plane and J.F. Ritt has given the classical account of the entire subject in his Integraion in Finite Terms, Columbia University Press, 1948. In spite of the essentially algebraic nature of the problem, all proofs so far have been analytic. This paper gives a self contained purely algebraic exposition of the probelm, making a few new points in addition to the resulting simplicity and generalization."

— axiom.bib —

@article{Rose72,

```
author = "Rosenlicht, Maxwell",
title = "Integration in finite terms",
journal = "American Mathematical Monthly",
year = "1972",
volume = "79",
pages = "963-972",
paper = "Rose72.pdf",
```

— ignore —

\bibitem[Rothstein 76]{Ro76} Rothstein, Michael

title = "Aspects of symbolic integration and simplification of exponential and primitive functi PhD thesis, University of Wisconsin-Madison (1976)

```
url = "http://www.cs.kent.edu/~rothstei/dis.pdf",
paper = "Ro76.pdf",
ref = "00051",
abstract = "
```

In this thesis we cover some aspects of the theory necessary to obtain a canonical form for functions obtained by integration and exponentiation from the set of rational functions.

These aspects include a new algorithm for symbolic integration of functions involving logarithms and exponentials which avoids factorization of polynomials in those cases where algebraic extension of the constant field is not required, avoids partial fraction decompositions, and only solves linear systems with a small number of unknowns.

We have also found a theorem which states, roughly speaking, that if integrals which can be represented as logarithms are represented as such, the only algebraic dependence that a new exponential or logarithm can satify is given by the law of exponents or the law of logarithms."

— ignore —

\bibitem[Rothstein 76a]{Ro76a} Rothstein, Michael; Caviness, B.F.
 title = "A structure theorem for exponential and primitive functions: a preliminary report",
ACM Sigsam Bulletin Vol 10 Issue 4 (1976)
 paper = "Ro76a.pdf",
 abstract = "

In this paper a generalization of the Risch Structure Theorem is reported. The generalization applies to fields $F(t_1,\ldots,t_n)$ where F is a differential field (in our applications F will be a finitely generated extension of Q, the field of rational numbers) and each t_i is either algebraic over $F_{i-1}=F(t_1,\ldots,t_{i-1})$, is an exponential of an element in F_{i-1} , or is an integral of an element in F_{i-1} . If t_i is an integral and can be expressed using logarithms, it must be so expressed for the generalized structure theorem to apply."

— ignore —

```
\bibitem[Rothstein 76b] {Ro76b} Rothstein, Michael; Caviness, B.F.
  title = "A structure theorem for exponential and primitive functions",
SIAM J. Computing Vol 8 No 3 (1979)
  paper = "Ro76b.pdf",
  ref = "00104",
  abstract = "
```

In this paper a new theorem is proved that generalizes a result of Risch. The new theorem gives all the possible algebraic relationships among functions that can be built up from the rational functions by algebraic operations, by taking exponentials, and by integration. The functions so generated are called exponential and primitive functions. From the theorem an algorithm for determining algebraic dependence among a given set of exponential and primitive functions is derived. The algorithm is then applied to a problem in computer algebra."

- ignore -

\bibitem[Seidenberg 58]{Sei58} Seidenberg, Abraham
 title = "Abstract differential algebra and the analytic case",
Proc. Amer. Math. Soc. Vol 9 pp159-164 (1958)

— ignore —

\bibitem[Seidenberg 69]{Sei69} Seidenberg, Abraham
 title = "Abstract differential algebra and the analytic case. II",
Proc. Amer. Math. Soc. Vol 23 pp689-691 (1969)

— ignore —

\bibitem[Singer 85]{Sing85} Singer, M.F.; Saunders, B.D.; Caviness, B.F.

title = "An extension of Liouville's theorem on integration in finite terms",

SIAM J. of Comp. Vol 14 pp965-990 (1985)

url = "http://www4 ncsu edu/~singer/papers/singer saunders caviness pdf"

url = "http://www4.ncsu.edu/~singer/papers/singer_saunders_caviness.pdf",
paper = "Sing85.pdf",
abstract = "

In Part 1 of this paper, we give an extension of Liouville's Theorem and give a number of examples which show that integration with special functions involves some phenomena that do not occur in integration with the elementary functions alone. Our main result generalizes Liouville's Theorem by allowing, in addition to the elementary functions, special functions such as the error function, Fresnel integrals and the logarithmic integral (but not the dilogarithm or exponential integral) to appear in the integral of an elementary function. The basic conclusion is that these functions, if they appear, appear linearly. We give an algorithm which decides if an elementary function, built up using only exponential functions and rational operations has an integral which can be expressed in terms of elementary functions and error functions."

— ignore —

\bibitem[Slagle 61]{Slag61} Slagle, J.

title = "A heuristic program that solves symbolic integration problems in freshman calculus", Ph.D Diss. MIT, May 1961; also Computers and Thought, Feigenbaum and Feldman. % REF:00014

— ignore —

\bibitem[Terelius 09]{Tere09} Terelius, Bjorn
title = "Symbolic Integration",
paper = "Tere09.pdf",
abstract = "

Symbolic integration is the problem of expressing an indefinite integral \hat{f} of a given function f as a finite combination g of elementary functions, or more generally, to determine whether a certain class of functions contains an element g such that $g^p = f$.

In the first part of this thesis, we compare different algorithms for symbolic integration. Specifically, we review the integration rules taught in calculus courses and how they can be used systematically to create a reasonable, but somewhat limited, integration method. Then we present the differential algebra required to prove the transcendental cases of Risch's algorithm. Risch's algorithm decides if the integral of an elementary function is elementary and if so computes it. The presentation is mostly self-contained and, we hope, simpler than previous descriptions of the algorithm. Finally, we describe Risch-Norman's algorithm which, although it is not a decision procedure, works well in practice and is considerably simpler than the full Risch algorithm.

In the second part of this thesis, we briefly discuss an implementation of a computer algebra system and some of the experiences it has given us. We also demonstrate an implementation of the rule-based approach and how it can be used, not only to compute integrals, but also to generate readable derivations of the results."

— axiom.bib —

@article{Trag76,
 author = "Trager, Barry",
 title = "Algebraic factoring and rational function integration",
 journal = "Proceedings of SYMSAC'76",
 year = "1976",
 pages = "219-226",

paper = "Trag76.pdf",
abstract = "

This paper presents a new, simple, and efficient algorithm for factoring polynomials in several variables over an algebraic number field. The algorithm is then used interatively to construct the splitting field of a polynomial over the integers. Finally the factorization and splitting field algorithms are applied to the problem of determining the transcendental part of the integral of a rational function. In particular, a constructive procedure is given for finding a least degree extension field in which the integral can be expressed."

— ignore —

\bibitem[Trager 76a]{Tr76a} Trager, Barry Marshall
 title = "Algorithms for Manipulating Algebraic Functions",
MIT Master's Thesis.

url = "http://www.dm.unipi.it/pages/gianni/public_html/Alg-Comp/fattorizzazione-EA.pdf",
paper = "Tr76a.pdf",
ref = "00050",

abstract = " Given a base field \$k\$, of characteristic zero, with effective procedures for performing arithmetic and factoring polynomials, this thesis presents algorithms for extending those capabilities to elements of a finite algebraic symbolic manipulation system. An algebraic factorization algorithm along with a constructive version of the primitive element theorem is used to construct splitting fields of polynomials. These fields provide a context in which we can operate symbolically with all the roots of a set of polynomials. One application for this capability is rational function integrations. Previously presented symbolic algorithms concentrated on finding the rational part and were only able to compute the complete integral in special cases. This thesis presents an algorithm for finding an algebraic extension field of least degreee in which the integral can be expressed, and then constructs the integral in that field. The problem of algebraic function integration is also examined, and a highly efficient procedure is presented for generating the algebraic part of integrals whose function fields are defined by a single radical extension of the rational functions."

```
@phdthesis{Trag84,
  author = "Trager, Barry",
  title = "On the integration of algebraic functions",
  school = "MIT",
  year = "1984",
  url = "http://www.dm.unipi.it/pages/gianni/public_html/Alg-Comp/thesis.pdf",
  paper = "Trag76.pdf",
  abstract = "
    We show how the "rational" approach for integrating algebraic
    functions can be extended to handle elementary functions. The
    resulting algorithm is a practical decision procedure for determining
    whether a given elementary function has an elementary antiderivative,
    and for computing it if it exists."
}
            — ignore —
\bibitem[W\"urfl 07]{Wurf07} W\"urfl, Andreas
  title = "Basic Concepts of Differential Algebra",
  url = "http://www14.in.tum.de/konferenzen/Jass07/courses/1/Wuerfl/wuerfl_paper.pdf",
  paper = "Wurf07.pdf",
  abstract = "
    Modern computer algebra systems symbolically integrate a vast variety
    of functions. To reveal the underlying structure it is necessary to
    understand infinite integration not only as an analytical problem but
    as an algebraic one. Introducing the differential field of elementary
    functions we sketch the mathematical tools like Liouville's Principle
    used in modern algorithms. We present Hermite's method for integration
    of rational functions as well as the Rothstein/Trager method for
    rational and for elementary functions. Further applications of the
    mentioned algorithms in the field of ODE's conclude this paper."
```

2.24 Partial Fraction Decomposition

```
-- ignore --
\bibitem[Angell] {Angell} Angell, Tom
  title = "Guidelines for Partial Fraction Decomposition",
  url = "http://www.math.udel.edu/~angell/partfrac_I.pdf",
  paper = "Angell.pdf",
```

```
— ignore —
\bibitem[Laval 08]{Lava08} Laval, Philippe B.
 title = "Partial Fractions Decomposition",
 url = "http://www.math.wisc.edu/~park/Fall2011/integration/Partial%20Fraction.pdf",
 paper = "Lava08.pdf",
           — ignore —
\bibitem[Mudd 14]{Mudd14} Harvey Mudd College
 title = "Partial Fractions",
  url = "http://www.math.hmc.edu/calculus/tutorials/partial_fractions/partial_fractions.pdf",
 paper = "Mudd14.pdf",
           — ignore —
\bibitem[Rajasekaran 14]{Raja14} Rajasekaran, Raja
 title = "Partial Fraction Expansion",
  url = "http://www.utdallas.edu/~raja1/EE4361%20Spring%2014/Lecture%20Notes/Partial%20Fractions
 paper = "Raja14.pdf",
           — ignore —
\bibitem[Wootton 14]{Woot14} Wootton, Aaron
 title = "Integration of Rational Functions by Partial Fractions",
  url = "http://faculty.up.edu/wootton/calc2/section7.4.pdf",
 paper = "Woot14.pdf",
```

2.25. ORE RINGS 111

2.25 Ore Rings

This is used as a reference for the LeftOreRing category, in particular, the least left common multiple (lcmCoef) function.

— ignore —

\bibitem[Abramov 97]{Abra97} Abramov, Sergei A.; van Hoeij, Mark
 title = "A method for the Integration of Solutions of Ore Equations",
Proc ISSAC 97 pp172-175 (1997)
 paper = "Abra97.pdf",
 abstract = "

We introduce the notion of the adjoint Ore ring and give a definition of adjoint polynomial, operator and equation. We apply this for integrating solutions of Ore equations."

— ignore —

\bibitem[Delenclos 06]{DL06} Delenclos, Jonathon; Leroy, Andr\'e
 title = "Noncommutative Symmetric functions and \$W\$-polynomials",
 url = "http://arxiv.org/pdf/math/0606614.pdf",
 paper = "DL06.pdf",
 abstract = "

Let \$K\$, \$D\$ be a division ring an endomorphism and a \$S\$-derivation of \$K\$, respectively. In this setting we introduce generalized noncommutative symmetric functions and obtain Vi\'ete formula and decompositions of different operators. \$W\$-polynomials show up naturally, their connetions with \$P\$-independency. Vandermonde and Wronskian matrices are briefly studied. The different linear factorizations of \$W\$-polynomials are analysed. Connections between the existence of LLCM (least left common multiples) of monic linear polynomials with coefficients in a ring and the left duo property are established at the end of the paper."

— ignore —

\bibitem[Abramov 05]{Abra05} Abramov, S.A.; Le, H.Q.; Li, Z.
''Univariate Ore Polynomial Rings in Computer Algebra''
url = "http://www.mmrc.iss.ac.cn/~zmli/papers/oretools.pdf",
paper = "Abra05.pdf",

abstract = "

We present some algorithms related to rings of Ore polynomials (or, briefly, Ore rings) and describe a computer algebra library for basic operations in an arbitrary Ore ring. The library can be used as a basis for various algorithms in Ore rings, in particular, in differential, shift, and \$q\$-shift rings."

2.26 Number Theory

```
@InProceedings{Kalt89d,
 author = "Kaltofen, E. and Valente, T. and Yui, N.",
 title = "An improved {Las Vegas} primality test",
 booktitle = "Proc. 1989 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC89",
 pages = "26--33",
 year = "1989",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/89/KVY89.pdf",
 paper = "Kalt89d.pdf",
            — axiom.bib —
@InCollection{Kalt91b,
 author = "Kaltofen, E. and Yui, N.",
  editor = "D. V. Chudnovsky and G. V. Chudnovsky and H. Cohn and
           M. B. Nathanson",
  title = "Explicit construction of {Hilbert} class fields of imaginary
           quadratic fields by integer lattice reduction",
 booktitle = "Number Theory New York Seminar 1989--1990",
  pages = "150--202",
 publisher = "Springer-Verlag",
 year = "1991",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/KaYui91.pdf",
 paper = "Kalt91b.pdf",
```

— axiom.bib — @InProceedings{Kalt84a, author = "Kaltofen, E. and Yui, N.", title = "Explicit construction of the {Hilbert} class field of imaginary quadratic fields with class number 7 and 11", booktitle = "Proc. EUROSAM '84", pages = "310--320", crossref = "EUROSAM84", year = "1984","http://www.math.ncsu.edu/~kaltofen/bibliography/84/KaYui84_eurosam.ps.gz", paper = "Kalt84a.ps", — ignore — \bibitem[Shoup 08]{Sho08} Shoup, Victor "A Computational Introduction to Number Theory" url = "http://shoup.net/ntb/ntb-v2.pdf", paper = "Sho08.pdf",

2.27 Sparse Polynomial Interpolation

```
— axiom.bib —
@InProceedings{Kalt07b,
  author = "Kaltofen, Erich and Yang, Zhengfeng",
  title = "On Exact and Approximate Interpolation of Sparse
           Rational Functions",
  year = "2007",
  booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'07",
  crossref = "ISSAC07",
  pages = "203--210",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/07/KaYa07.pdf",
  paper = "Kalt07b.pdf",
            — axiom.bib —
@Article{Gies03,
  author = "Giesbrecht, Mark and Kaltofen, Erich and Lee, Wen-shin",
  title = "Algorithms for Computing Sparsest Shifts of Polynomials in
           Power, {Chebychev}, and {Pochhammer} Bases",
  year = "2003",
  journal = "Journal of Symbolic Computation",
  volume = "36",
  number = "3--4",
  pages = "401--424",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/03/GKL03.pdf",
  paper = "Gies03.pdf",
}
            — axiom.bib —
@InProceedings{Gies02,
  author = "Giesbrecht, Mark and Kaltofen, Erich and Lee, Wen-shin",
  title = "Algorithms for Computing the Sparsest Shifts for Polynomials via the
           {Berlekamp}/{Massey} Algorithm",
  booktitle = "Proc. 2002 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC02",
  pages = "101--108",
  year = "2002",
```

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url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/GKL02.pdf",
 paper = "Gies02.pdf",
            — axiom.bib —
@Article{Kalt03b,
  author = "Kaltofen, Erich and Lee, Wen-shin",
 title = "Early Termination in Sparse Interpolation Algorithms",
 year = "2003",
  journal = "Journal of Symbolic Computation",
 volume = "36",
 number = "3--4",
 pages = "365--400",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/03/KL03.pdf",
 paper = "Kalt03b.pdf",
            — axiom.bib —
@InProceedings{Kalt00a,
 author = "Kaltofen, E. and Lee, W.-s. and Lobo, A.A.",
 title = "Early termination in {Ben-Or/Tiwari} sparse interpolation
          and a hybrid of {Zippel}'s algorithm",
 booktitle = "Proc. 2000 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC2K",
 pages = "192--201",
 year = "2000",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/2K/KLL2K.pdf",
 paper = "Kalt00a.pdf",
            — axiom.bib —
@InProceedings{Kalt10b,
 author = "Kaltofen, Erich L.",
 title = "Fifteen years after {DSC} and {WLSS2} {What} parallel
```

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computations {I} do today [{Invited} Lecture at {PASCO} 2010]",
  year = "2010",
  booktitle = "Proc. 2010 Internat. Workshop on Parallel Symbolic Comput.",
  crossref = "PASCO10",
  pages = "10--17",
  month = "July",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/10/Ka10_pasco.pdf",
 paper = "Kalt10b.pdf",
            — axiom.bib —
@InProceedings{Kalt90,
  author = "Kaltofen, E. and Lakshman, Y.N. and Wiley, J.M.",
  editor = "S. Watanabe and M. Nagata",
  title = "Modular rational sparse multivariate polynomial interpolation",
  booktitle = "Proc. 1990 Internat. Symp. Symbolic Algebraic Comput.",
  pages = "135--139",
  publisher = "ACM Press",
  year = "1990",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/90/KLW90.pdf",
 paper = "Kalt90.pdf",
}
            — axiom.bib —
@InProceedings{Kalt88a,
  author = "Kaltofen, E. and Yagati, Lakshman",
  title = "Improved sparse multivariate polynomial interpolation algorithms",
  booktitle = "Symbolic Algebraic Comput. Internat. Symp. ISSAC '88 Proc.",
  crossref = "ISSAC88",
  pages = "467--474",
  year = "1988",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/88/KaLa88.pdf",
  paper = "Kalt88a.pdf",
}
```

2.28 Divisions and Algebraic Complexity

```
— axiom.bib —
@InCollection{Gren11,
  author = "Grenet, Bruno and Kaltofen, Erich L. and Koiran, Pascal
           and Portier, Natacha",
  title = "Symmetric Determinantal Representation of Formulas and Weakly
           Skew Circuits",
  booktitle = "Randomization, Relaxation, and Complexity in Polynomial
               Equation Solving",
  year = "2011",
  editor = "Leonid Gurvits and Philippe P\'{e}bay and J. Maurice Rojas
            and David Thompson",
  pages = "61--96",
  publisher = "American Mathematical Society",
  address = "Providence, Rhode Island, USA",
  isbn = "978-0-8218-5228-6",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/10/GKKP10.pdf",
 paper = "Gren11.pdf",
            — axiom.bib —
@InProceedings{Kalt08a,
  author = "Kaltofen, Erich and Koiran, Pascal",
  title = "Expressing a Fraction of Two Determinants as a Determinant",
  year = "2008",
  booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'08",
  crossref = "ISSAC08",
 pages = "141--146",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/08/KaKoi08.pdf",
 paper = "Kalt08a.pdf",
            — axiom.bib —
@Article{Hitz95,
  author = "Kitz, M.A. and Kaltofen, E.",
  title = "Integer division in residue number systems",
```

```
journal = "IEEE Trans. Computers",
  year = "1995",
  volume = "44",
  number = "8",
  pages = "983--989",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/HiKa95.pdf",
 paper = "Hitz95.pdf",
}
            — axiom.bib —
@InProceedings{Kalt92a,
  author = "Kaltofen, E.",
  title = "On computing determinants of matrices without divisions",
  booktitle = "Proc. 1992 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC92",
  pages = "342--349",
 year = "1992",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/92/Ka92_issac.pdf",
 paper = "Kalt92a.pdf",
            — axiom.bib —
@Article{Cant91,
  author = "Cantor, D.G. and Kaltofen, E.",
  title = "On fast multiplication of polynomials over arbitrary algebras",
  journal = "Acta Inform.",
  year = "1991",
  volume = "28",
  number = "7",
  pages = "693--701",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/CaKa91.pdf",
  paper = "Cant91.pdf",
            — axiom.bib —
```

2.29 Polynomial Factorization

```
— axiom.bib —
@PhdThesis{Kalt82,
  author = "Kaltofen, E.",
 title = "On the complexity of factoring polynomials with integer
          coefficients",
  school = "RPI",
  address = "Troy, N. Y.",
 year = "1982",
 month = "December",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/82/Ka82_thesis.pdf",
 paper = "Kalt82.pdf",
            — axiom.bib —
@Article{Gath85,
 author = "{von zur Gathen}, Joachim and Kaltofen, E.",
  title = "Factoring sparse multivariate polynomials",
  journal = "J. Comput. System Sci.",
 year = "1985",
 volume = "31",
 pages = "265--287",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/85/GaKa85_mathcomp.ps.gz",
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paper = "Gath85.ps",
           — axiom.bib —
@InCollection{Kalt11c,
 author = "Kaltofen, Erich and Lecerf, Gr{\'e}goire",
 title = "Section 11.5. {Factorization} of multivariate polynomials",
 booktitle = "Handbook of Finite Fields",
 crossref = "HFF11",
 pages = "382--392",
 year = "2011",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/11/KL11.pdf",
 paper = "Kalt11c.pdf",
           — axiom.bib —
@InProceedings{Kalt05b,
  author = "Kaltofen, Erich and Koiran, Pascal",
  title = "On the complexity of factoring bivariate supersparse
           (lacunary) polynomials",
  year = "2005",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'05",
 crossref = "ISSAC05",
 pages = "208--215",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/05/KaKoi05.pdf",
 paper = "Kalt05b.pdf",
           — axiom.bib —
@InProceedings{Kalt06a,
 author = "Kaltofen, Erich and Koiran, Pascal",
  title = "Finding Small Degree Factors of Multivariate Supersparse
           (Lacunary) Polynomials Over Algebraic Number Fields",
  year = "2006",
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booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'06",
  crossref = "ISSAC06",
 pages = "162--168",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/06/KaKoi06.pdf",
 paper = "Kalt06a.pdf",
            — axiom.bib —
@InProceedings{Kalt97a,
  author = "Kaltofen, E. and Shoup, V.",
 title = "Fast polynomial factorization over high algebraic extensions of
          finite fields",
 booktitle = "Proc. 1997 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC97",
 year = "1997",
 pages = "184--188",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/97/KaSh97.pdf",
 paper = "Kalt97a.pdf",
            — axiom.bib —
@Article{Kalt98,
  author = "Kaltofen, E. and Shoup, V.",
 title = "Subquadratic-time factoring of polynomials over finite fields",
  journal = "Math. Comput.",
 month = "July",
  year = "1998",
  volume = "67",
 number = "223",
 pages = "1179--1197",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/98/KaSh98.pdf",
 paper = "Kalt98.pdf",
            — axiom.bib —
```

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@InProceedings{Kalt95a,
  author = "Kaltofen, E. and Shoup, V.",
  title = "Subquadratic-time factoring of polynomials over finite fields",
 booktitle = "Proc. 27th Annual ACM Symp. Theory Comput.",
  year = "1995",
 publisher = "ACM Press",
 address = "New York, N.Y.",
 pages = "398--406",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/KaSh95.ps.gz",
 paper = "Kalt95a.ps",
            — axiom.bib —
@InProceedings{Diaz95,
  author = "Diaz, A. and Kaltofen, E.",
  title = "On computing greatest common divisors with polynomials given by
           black boxes for their evaluation",
 booktitle = "Proc. 1995 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC95",
 pages = "232--239",
 year = "1995",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/DiKa95.ps.gz",
 paper = "Diaz95.ps",
            — axiom.bib —
@InProceedings{Kalt88,
  author = "Kaltofen, E. and Trager, B.",
  title = "Computing with polynomials given by black boxes for their
    evaluations: Greatest common divisors, factorization, separation of
   numerators and denominators",
 booktitle = "Proc. 29th Annual Symp. Foundations of Comp. Sci.",
 pages = "296--305",
 year = "1988",
 organization = "IEEE",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/88/focs88.ps.gz",
 paper = "Kalt88.ps",
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paper = "Kalt87b.pdf",

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— axiom.bib —
@InProceedings{Kalt85b,
  author = "Kaltofen, E.",
  title = "Computing with polynomials given by straight-line programs {II};
          sparse factorization",
  booktitle = "Proc. 26th Annual Symp. Foundations of Comp. Sci.",
 year = "1985",
 pages = "451--458",
 organization = "IEEE",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_focs.ps.gz",
 paper = "Kalt85b.ps",
            — axiom.bib —
@InProceedings{Kalt86,
  author = "Kaltofen, E.",
  title = "Uniform closure properties of p-computable functions",
  booktitle = "Proc. 18th Annual ACM Symp. Theory Comput.",
  year = "1986",
 pages = "330--337",
  organization = "ACM",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/86/Ka86_stoc.pdf",
 paper = "Kalt86.pdf",
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            — axiom.bib —
@InProceedings{Kalt87b,
  author = "Kaltofen, E.",
  title = "Single-factor Hensel lifting and its application to the
          straight-line complexity of certain polynomials",
 booktitle = "Proc. 19th Annual ACM Symp. Theory Comput.",
  year = "1987",
 pages = "443--452",
 organization = "ACM",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/87/Ka87_stoc.pdf",
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@Article{Kalt87c,

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}
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@InCollection{Kalt89,
  author = "Kaltofen, E.",
  editor = "S. Micali",
  title = "Factorization of polynomials given by straight-line programs",
  booktitle = "Randomness and Computation",
  pages = "375--412",
  publisher = "JAI Press Inc.",
  year = "1989",
  volume = "5",
  series = "Advances in Computing Research",
  address = "Greenwhich, Connecticut",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/89/Ka89_slpfac.pdf",
 paper = "Kalt89.pdf",
            — axiom.bib —
@Article{Gao04,
  author = "Gao, Shuhong and Kaltofen, E. and Lauder, A.",
  title = "Deterministic distinct degree factorization for polynomials
           over finite fields",
  year = "2004",
  journal = "Journal of Symbolic Computation",
  volume = "38",
  number = "6",
  pages = "1461--1470",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/01/GKL01.pdf",
 paper = "GaoO4.pdf",
            — axiom.bib —
```

```
author = "Kaltofen, E.",
  title = "Deterministic irreducibility testing of polynomials over
          large finite fields",
  journal = "Journal of Symbolic Computation",
 year = "1987",
  volume = "4",
 pages = "77--82",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/87/Ka87_jsc.ps.gz",
 paper = "Kalt87c.ps",
            — axiom.bib —
@Article{Kalt95b,
  author = "Kaltofen, E.",
 title = "Effective {Noether} irreducibility forms and applications",
  journal = "J. Comput. System Sci.",
  year = "1995",
  volume = "50",
 number = "2",
 pages = "274--295",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/95/Ka95_jcss.pdf",
 paper = "Kalt95b.pdf",
            — axiom.bib —
@Article{Kalt85a,
  author = "Kaltofen, E.",
 title = "Fast parallel absolute irreducibility testing",
  journal = "Journal of Symbolic Computation",
  year = "1985",
  volume = "1",
 number = "1",
 pages = "57--67",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_jsc.pdf",
 paper = "Kalt85a.pdf",
}
```

```
@Article{Gath85a,
 author = "{von zur Gathen}, Joachim and Kaltofen, E.",
 title = "Factoring multivariate polynomials over finite fields",
 journal = "Math. Comput.",
 year = "1985",
 volume = "45",
 pages = "251--261",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/85/GaKa85_mathcomp.ps.gz",
 paper = "Gath85a.ps",
            — axiom.bib —
@Article{Kalt85e,
  author = "Kaltofen, E.",
  title = "Polynomial-time reductions from multivariate to bi- and univariate
          integral polynomial factorization",
  journal = "{SIAM} J. Comput.",
 year = "1985",
 volume = "14",
 number = "2",
 pages = "469--489",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_sicomp.pdf",
 paper = "Kalt85e.pdf",
}
            — axiom.bib —
@InProceedings{Kalt82a,
  author = "Kaltofen, E.",
  title = "A polynomial-time reduction from bivariate to univariate
           integral polynomial factorization",
 booktitle = "Proc. 23rd Annual Symp. Foundations of Comp. Sci.",
  year = "1982",
  pages = "57--64",
  organization = "IEEE",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/82/Ka82_focs.pdf",
```

@InCollection{Kalt90c,

```
paper = "Kalt82a.pdf",
            — axiom.bib —
@InProceedings{Kalt03,
 author = "Kaltofen, Erich",
 title = "Polynomial Factorization: a Success Story",
 year = "2003",
 booktitle = "Symbolic Algebraic Comput. Internat. Symp. ISSAC '88 Proc.",
  crossref = "ISSAC03",
 pages = "3--4",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/03/Ka03.pdf",
 keywords = "survey",
 paper = "Kalt03.pdf",
}
            — axiom.bib —
@InProceedings{Kalt92b,
 author = "Kaltofen, E.",
 title = "Polynomial factorization 1987-1991",
 booktitle = "Proc. LATIN '92",
  editor = "I. Simon",
  series = "Lect. Notes Comput. Sci.",
  volume = "583",
 pages = "294--313",
 publisher = "Springer-Verlag",
 year = "1992",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/92/Ka92_latin.pdf",
 keywords = "survey",
 paper = "Kalt92b.pdf",
            — axiom.bib —
```

```
author = "Kaltofen, E.",
  editor = "D. V. Chudnovsky and R. D. Jenks",
  title = "Polynomial Factorization 1982-1986",
  booktitle = "Computers in Mathematics",
  pages = "285--309",
  publisher = "Marcel Dekker, Inc.",
  year = "1990",
  volume = "125",
  series = "Lecture Notes in Pure and Applied Mathematics",
  address = "New York, N. Y.",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/90/Ka90_survey.ps.gz",
 keywords = "survey",
 paper = "Kalt90c.ps",
            — axiom.bib —
@InCollection{Kalt82b,
  author = "Kaltofen, E.",
  title = "Polynomial factorization",
  editor = "B. Buchberger and G. Collins and R. Loos",
 booktitle = "Computer Algebra",
  edition = "2",
  pages = "95--113",
 publisher = "Springer-Verlag",
 year = "1982",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/82/Ka82_survey.ps.gz",
 keywords = "survey",
 paper = "Kalt82b.ps",
```

2.30 Branch Cuts

```
@article{Beau03,
   author = "Beaumont, James and Bradford, Russell and Davenport, James H.",
   title = "Better simplification of elementary functions through power series",
   journal = "2003 International Symposium on Symbolic and Algebraic Computation",
   series = "ISSAC'03",
```

```
year = "2003",
  month = "August",
  paper = "Beau03.pdf",
  abstract = "
    In [5], we introduced an algorithm for deciding whether a proposed
    simplification of elementary functions was correct in the presence of
    branch cuts. This algorithm used multivalued function simplification
    followed by verification that the branches were consistent.
    In [14] an algorithm was presented for zero-testing functions defined
    by ordinary differential equations, in terms of their power series.
    The purpose of the current paper is to investigate merging the two
    techniques. In particular, we will show an explicit reduction to the
    constant problem [16]."
}
            — axiom.bib —
@article{Beau07,
  author = "Beaumont, James C. and Bradford, Russell J. and
            Davenport, James H. and Phisanbut, Nalina",
  title = "Testing elementary function identities using CAD",
  journal = "Applicable Algebra in Engineering, Communication and Computing",
  year = "2007",
  volume = "18",
  number = 6,
  issn = "0938-1279",
  publisher = "Springer-Verlag",
  pages = "513-543",
  paper = "Beau07.pdf",
  abstract = "
    One of the problems with manipulating function identities in computer
    algebra systems is that they often involve functions which are
    multivalued, whilst most users tend to work with single-valued
    functions. The problem is that many well-known identities may no
    longer be true everywhere in the complex plane when working with their
    single-valued counterparts. Conversely, we cannot ignore them, since
    in particular contexts they may be valid. We investigate the
```

practicality of a method to verify such identities by means of an experiment; this is based on a set of test examples which one might realistically meet in practice. Essentially, the method works as follows. We decompose the complex plane via means of cylindrical algebraic decomposition into regions with respect to the branch cuts of the functions. We then test the identity numerically at a sample point in the region. The latter step is facilitated by the notion of

the {\sl adherence} of a branch cut, which was previously introduced by the authors. In addition to presenting the results of the experiment, we explain how adherence relates to the proposal of {\sl signed zeros} by W. Kahan, and develop this idea further in order to allow us to cover previously untreatable cases. Finally, we discuss other ways to improve upon our general methodology as well as topics for future research."

}

— axiom.bib —

@article{Brad02,

author="Bradford, Russell and Corless, Robert M. and Davenport, James H. and Jeffrey, David J. and Watt, Stephen M.", title="Reasoning about the Elementary Functions of Complex Analysis", journal="Annals of Mathematics and Artificial Intelligence", year="2002", issn="1012-2443", volume="36", number="3", doi="10.1023/A:1016007415899", url="http://dx.doi.org/10.1023/A%3A1016007415899", publisher="Kluwer Academic Publishers", keywords="elementary functions; branch cuts; complex identities", pages="303-318", paper = "Brad02.pdf", abstract = " There are many problems with the simplification of elementary functions, particularly over the complex plane, though not

functions, particularly over the complex plane, though not exclusively. Systems tend to make ''howlers'' or not to simplify enough. In this paper we outline the ''unwinding number'' approach to such problems, and show how it can be used to prevent errors and to systematise such simplification, even though we have not yet reduced the simplification process to a complete algorithm. The unsolved problems are probably more amenable to the techniques of artificial intelligence and theorem proving than the original problem of complex variable analysis."

}

— axiom.bib —

@inproceedings{Chyz11,

```
author = "Chyzak, Fr{\'e}d{\'e}ric and Davenport, James H. and
            Koutschan, Christoph and Salvy, Bruno",
  title = "On Kahan's Rules for Determining Branch Cuts",
  booktitle = "Proc. 13th Int. Symp. on Symbolic and Numeric Algorithms for Scientific Computing",
  year = "2011",
  isbn = "978-1-4673-0207-4",
  location = "Timisoara",
  pages = "47-51",
  doi = "10.1109/SYNASC.2011.51",
  acmid = "258794",
  publisher = "IEEE"
 paper = "Chyz11.pdf",
  abstract = "
    In computer algebra there are different ways of approaching the
    mathematical concept of functions, one of which is by defining them as
    solutions of differential equations. We compare different such
    appraoches and discuss the occurring problems. The main focus is on
    the question of determining possible branch cuts. We explore the
    extent to which the treatment of branch cuts can be rendered (more)
    algorithmic, by adapting Kahan's rules to the differential equation
    setting."
}
            — axiom.bib —
@article{Dave10,
  author = "Davenport, James",
  title = {The Challenges of Multivalued "Functions"},
  journal = "Lecture Notes in Computer Science",
  volume = "6167",
  year = "2010",
  pages = "1-12",
  paper = "Dave10.pdf",
  abstract = "
    Although, formally, mathematics is clear that a function is a
    single-valued object, mathematical practice is looser, particularly
    with n-th roots and various inverse functions. In this paper, we point
    out some of the looseness, and ask what the implications are, both for
    Artificial Intelligence and Symbolic Computation, of these practices.
    In doing so, we look at the steps necessary to convert existing tests
    into
    \begin{itemize}
    \item (a) rigorous statements
    \item (b) rigorously proved statements
    \end{itemize}
```

In particular we ask whether there might be a constant "'de Bruij factor"

}

[18] as we make these texts more formal, and conclude that the answer depends greatly on the interpretation being placed on the symbols."

— axiom.bib —

@article{Dave12,

author = "Davenport, James H. and Bradford, Russell and England, Matthew and Wilson, David",

year = "2012",
series = "SYNASC'12",
pages = "83-88",
publisher = "IEEE",
paper = "Dave12.pdf",
abstract = "

In considering the reliability of numerical programs, it is normal to "'limit our study to the semantics dealing with numerical precision''. On the other hand, there is a great deal of work on the reliability of programs that essentially ignores the numerics. The thesis of this paper is that there is a class of problems that fall between the two, which could be described as "'does the low-level arithmetic implement the high-level mathematics''. Many of these problems arise because mathematics, particularly the mathematics of the complex numbers, is more difficult than expected; for example the complex function log is not continuous, writing down a program to compute an inverse function is more complicated than just solving an equation, and many algebraic simplification rules are not universally valid.

The good news is that these problems are theoretically capable of being solved, and are practically close to being solved, but not yet solved, in several real-world examples. However, there is still a long way to go before implementations match the theoretical possibilities."

— axiom.bib —

@article{Jeff04, author = "Jeffrey, D. J. and Norman, A. C.",

```
title = "Not Seeing the Roots for the Branches: Multivalued Functions in
           Computer Algebra",
  journal = "SIGSAM Bull.",
  issue_date = "September 2004",
  volume = "38".
  number = "3",
  month = "September",
  year = "2004",
  issn = "0163-5824",
 pages = "57--66",
  numpages = "10",
  url = "http://doi.acm.org/10.1145/1040034.1040036",
  doi = "10.1145/1040034.1040036",
  acmid = "1040036",
  publisher = "ACM",
  address = "New York, NY, USA",
  paper = "Jeff04.pdf",
  abstract = "
    We discuss the multiple definitions of multivalued functions and their
    suitability for computer algebra systems. We focus the discussion by
    taking one specific problem and considering how it is solved using
    different definitions. Our example problem is the classical one of
    calculating the roots of a cubic polynomial from the Cardano formulae,
    which contains fractional powers. We show that some definitions of
    these functions result in formulae that are correct only in the sense
    that they give candidates for solutions; these candidates must then be
    tested. Formulae that are based on single-valued functions, in
    contract, are efficient and direct."
}
            — axiom.bib —
@inproceedings{Kaha86,
  author = "Kahan, W.",
  title = "Branch cuts for complex elementary functions",
 booktitle = "The State of the Art in Numerical Analysis",
  year = "1986",
 month = "April",
 editor = "Powell, M.J.D and Iserles, A.",
  publisher = "Oxford University Press"
}
```

— axiom.bib —

```
@article{Rich96,
  author = "Rich, Albert D. and Jeffrey, David J.",
  title = "Function Evaluation on Branch Cuts",
  journal = "SIGSAM Bull.",
  issue_date = "June 1996",
  volume = "30",
 number = "2",
 month = "June",
  year = "1996",
  issn = "0163-5824",
  pages = "25--27",
  numpages = "3",
  url = "http://doi.acm.org/10.1145/235699.235704",
  doi = "10.1145/235699.235704",
  acmid = "235704",
  publisher = "ACM",
  address = "New York, NY, USA",
  abstract = "
    Once it is decided that a CAS will evaluate multivalued functions on
    their principal branches, questions arise concerning the branch
    definitions. The first questions concern the standardization of the
   positions of the branch cuts. These questions have largely been
    resolved between the various algebra systems and the numerical
    libraries, although not completely. In contrast to the computer
    systems, many mathematical textbooks are much further behind: for
    example, many popular textbooks still specify that the argument of a
    complex number lies between 0 and 2\pi. We do not intend to discuss
    these first questions here, however. Once the positions of the branch
    cuts have been fixed, a second set of questions arises concerning the
    evaluation of functions on their branch cuts."
}
```

```
@article{Patt96,
   author = "Patton, Charles M.",
   title = "A Representation of Branch-cut Information",
   journal = "SIGSAM Bull.",
   issue_date = "June 1996",
   volume = "30",
   number = "2",
   month = "June",
   year = "1996",
```

}

```
issn = "0163-5824",
  pages = "21--24",
  numpages = "4",
  url = "http://doi.acm.org/10.1145/235699.235703",
 doi = "10.1145/235699.235703",
  acmid = "235703",
  publisher = "ACM",
  address = "New York, NY, USA",
 paper = "Patt96.pdf",
  abstract = "
    Handling (possibly) multi-valued functions is a problem in all current
    computer algebra systems. The problem is not an issue of technology.
    Its solution, however, is tied to a uniform handling of the issues by
    the mathematics community."
}
            — axiom.bib —
@article{Squi91,
  author = "Squire, Jon S.",
  title = "Rationale for the Proposed Standard for a Generic Package of
           Complex Elementary Functions",
  journal = "Ada Lett.",
  issue_date = "Fall 1991",
  volume = "XI",
  number = "7",
 month = "September",
  year = "1991",
  issn = "1094-3641",
  pages = "166--179",
  numpages = "14",
  url = "http://doi.acm.org/10.1145/123533.123545",
  doi = "10.1145/123533.123545",
  acmid = "123545",
  publisher = "ACM",
  address = "New York, NY, USA",
  paper = "Squi91.pdf",
  abstract = "
    This document provides the background on decisions that were made
    during the development of the specification for Generic Complex
    Elementary fuctions. It also rovides some information that was used to
    develop error bounds, range, domain and definitions of complex
    elementary functions."
```

```
@article{Squi91a,
  editor = "Squire, Jon S.",
  title = "Proposed Standard for a Generic Package of Complex
          Elementary Functions",
  journal = "Ada Lett.",
  issue_date = "Fall 1991",
  volume = "XI",
  number = "7",
  month = "September",
  year = "1991",
  issn = "1094-3641",
  pages = "140--165",
  numpages = "26",
  url = "http://doi.acm.org/10.1145/123533.123544",
  doi = "10.1145/123533.123544",
  acmid = "123544",
  publisher = "ACM",
  address = "New York, NY, USA",
  abstract = "
    This document defines the specification of a generic package of
    complex elementary functions called Generic Complex Elementary
    Functions. It does not provide the body of the package."
}
```

2.31 Square-free Decomposition

- axiom.bib -

```
paper = "Bern97.pdf",
abstract = "
```

}

In this paper we present a new deterministic algorithm for computing the square-free decomposition of multivariate polynomials with coefficients from a finite field.

Our algorithm is based on Yun's square-free factorization algorithm for characteristic O. The new algorithm is more efficient than existing, deterministic algorithms based on Musser's squarefree algorithm

We will show that the modular approach presented by Yun has no significant performance advantage over our algorithm. The new algorithm is also simpler to implement and it can rely on any existing GCD algorithm without having to worry about choosing ''good'' evaluation points.

To demonstrate this, we present some timings using implementations in Maple (Char et al. 1991), where the new algorithm is used for Release 4 onwards, and Axiom (Jenks and Sutor, 1992) which is the only system known to the author to use and implementation of Yun's modular algorithm mentioned above."

```
@article{Chez07,
  author = "Ch\'eze, Guillaume and Lecerf, Gr\'egoire",
  title = "Lifting and recombination techniques for absolute factorization",
  journal = "Journal of Complexity",
  volume = "23",
  number = "3",
  vear = "2007",
  month = "June",
  pages = "380-420",
  paper = "Chez07.pdf",
  abstract = "
    In the vein of recent algorithmic advances in polynomial factorization
    based on lifting and recombination techniques, we present new faster
    algorithms for computing the absolute factorization of a bivariate
    polynomial. The running time of our probabilistic algorithm is less
    than quadratic in the dense size of the polynomial to be factored."
```

— axiom.bib —

```
@article{Lece07,
  author = "Lecerf, Gr\', egoire",
  title = "Improved dense multivariate polynomial factorization algorithms",
  journal = "Journal of Symbolic Computation",
  volume = "42",
  number = "4",
  year = "2007",
  month = "April",
  pages = "477-494"
  paper = "Lece07.pdf",
  abstract = "
    We present new deterministic and probabilistic algorithms that reduce
    the factorization of dense polynomials from several variables to one
    variable. The deterministic algorithm runs in sub-quadratic time in
    the dense size of the input polynomial, and the probabilistic
    algorithm is softly optimal when the number of variables is at least
    three. We also investigate the reduction from several to two variables
    and improve the quantitative versions of Bertini's irreducibility theorem."
}
```

— axiom.bib —

```
@article{Wang77,
   author = "Wang, Paul S.",
   title = "An efficient squarefree decomposition algorithm",
   journal = "ACM SIGSAM Bulletin",
   volume = "11",
   number = "2",
   year = "1977",
   month = "May",
   pages = "4-6",
   paper = "Wang77.pdf",
   abstract = "
The concept of polynomial squarefree decomposition is an
```

The concept of polynomial squarefree decomposition is an important one in algebraic computation. The squarefree decomposition process has many uses in computer symbolic computation. A recent survey by D. Yun [3] describes many useful algorithms for this purpose. All of these methods depend on computing the greated common divisor (gcd) of the polynomial to be decomposed and its first derivative (with repect to some variable). In the multivariate case, this gcd computation is non-trivial and dominates the cost for the squarefree decompostion."

```
@article{Wang79,
  author = "Wang, Paul S. and Trager, Barry M.",
  title = "New Algorithms for Polynomial Square-Free Decomposition
          over the Integers",
  journal = "SIAM Journal on Computing",
  volume = "8",
  number = "3",
  year = "1979",
  publisher = "Society for Industrial and Applied Mathematics",
  issn = "00975397",
  paper = "Wang79.pdf",
  abstract = "
    Previously known algorithms for polynomial square-free decomposition
    rely on greatest common divisor (gcd) computations over the same
    coefficient domain where the decomposition is to be performed. In
    particular, gcd of the given polynomial and its first derivative (with
    respect to some variable) is obtained to begin with. Application of
    modular homomorphism and $p$-adic construction (multivariate case) or
    the Chinese remainder algorithm (univariate case) results in new
    square-free decomposition algorithms which, generally speaking, take
    less time than a single gcd between the given polynomial and its first
    derivative. The key idea is to obtain one or several "correct"
    homomorphic images of the desired square-free decomposition
    first. This provides information as to how many different square-free
    factors there are, their multiplicities and their homomorphic
    images. Since the multiplicities are known, only the square-free
    factors need to be constructed. Thus, these new algorithms are
    relatively insensitive to the multiplicities of the square-free factors."
}
```

```
@inproceedings{Yun76,
  author = "Yun, D.Y.Y",
  title = "On square-free decomposition algorithms",
  booktitle = "Proceedings of SYMSAC'76",
  year = "1976",
  keywords = "survey",
  pages = "26-35"
}
```

2.32 Symbolic Summation

— axiom.bib —

or, equivalently, by

```
@article{Abra71,
  author = "Abramov, S.A.",
  title = "On the summation of rational functions",
  year = "1971",
  journal = "USSR Computational Mathematics and Mathematical Physics",
  volume = "11",
  number = "4",
  pages = "324--330",
  paper = "Abra71.pdf",
  abstract = "
    An algorithm is given for solving the following problem: let
    F(x_1,\ldots,x_n) be a rational function of the variables
    $x_i$ with rational (read or complex) coefficients; to see if
    there exists a rational function G(v,w,x_2,\ldots,x_n) with
    coefficients from the same field, such that
    \[ \sum_{x_1=v}^w \{F(x_1, \ldots, x_n)\} = G(v, w, x_2, \ldots, x_n) \]
    for all integral values of $v \le w$. If $G$ exists, to obtain it.
    Realization of the algorithm in the LISP language is discussed."
}
            — axiom.bib —
@article{Gosp78,
  author = "Gosper, R. William",
  title = "Decision procedure for indefinite hypergeometric summation",
  year = "1978",
  journal = "Proc. Natl. Acad. Sci. USA",
  volume = "75",
  number = "1",
  pages = "40--42",
  month = "January",
  paper = "Gosp78.pdf",
    Given a summand $a_n$, we seek the 'indefinite sum' $S(n)$
    determined (within an additive constant) by
    [\sum_{n=1}^m{a_n} = S(m)=S(0)]
```

```
[a_n=S(n)-S(n-1)]
                An algorithm is exhibited which, given $a_n$, finds those $S(n)$
                with the property
                \[ \displaystyle \frac{S(n)}{S(n-1)} = \textrm{a rational function of n} \]
                With this algorithm, we can determine, for example, the three
                identities
                \[ \] =1 ^m{
                \frac{\displaystyle \int_{j=1}^{n-1}{bj^2+cj+d}}
                {\displaystyle\prod_{j=1}^n{bj^2+cj+e}}=
                \[ \displaystyle \sum_{n=1}^m {
                \frac{\displaystyle \frac{j=1}^{n-1}}{aj^3+bj^2+cj+d}}
                                    {\displaystyle \{ \Big\{ j=1 \}^n \{ aj^3+bj^2+cj+e \} \} = \} }
                \frac{1-{\displaystyle \frac{1-{\displaystyle \frac{j=1}^m}}}}{}
                \frac{aj^3+bj^2+cj+d}{aj^3+bj^2+cj+e}}{e-d}}\]
                \[ \] =1 ^m{
                {\displaystyle \{ \Big\{ j=1 \}^{n+1} \} \} } = {\displaystyle \{ bj^2+cj+e \} } = {\displaystyle \{ bj^2+cj+e \} } 
                \displaystyle\frac{
                \displaystyle\frac{2b}{e-d}-
                \label{lem:displaystyle} $$ \arrowvert a constraint of the const
                \left(
                \label{lem:displaystyle} $$  \displaystyle\frac{2b}{e-d}-\frac{b(2m+3)+c+d-e}{b(m+1)^2+c(m+1)+e} $$
                \right)
                \label{linear_displaystyle} $$ \bigg\{ j=1 ^m{\frac{bj^2+cj+d}{bj^2+cj+e}} \
                b^2-c^2+d^2+e^2+2bd-2de+2eb}
}
                                                — axiom.bib —
@article{Karr81,
        author = "Karr, Michael",
        title = "Summation in Finite Terms",
        journal = "Journal Association for Computing Machinery",
        year = "1981",
        volume = "28",
        number = "2",
        month = "April",
        issn = "0004-5411",
        pages = "305--350",
        url = "http://doi.acm.org/10.1145/322248.322255",
        publisher = "ACM",
       paper = "Karr81",
        abstract = "
                Results which allow either the computation of symbolic solutions to
                first-order linear difference equations or the determination that
```

}

solutions of a certain form do not exist are presented. Starting with a field of constants, larger fields may be constructed by the formal adjunction of symbols which behave like solutions to first-order linear equations (with a few restrictions). It is in these extension fields that the difference equations may be posed and in which the solutions are requested. The principal application of these results is in finding formulas for a broad class of finite sums or in showing the nonexistence of such formula."

— axiom.bib —

```
@book{Lafo82,
   author = "Lafon, J.C.",
   title = "Summation in Finite Terms",
   year = "1982",
   publisher = "Springer-Verlag",
   isbn = "3-211-81776-X",
   pages = "71--77",
   keywords = "axiomref,survey",
   abstract = "
    A survey on algorithms for summation in finite terms is given. After a
   precise definition of the problem the cases of polynomial and rational
   summands are treated. The main concern of this paper is a description
   of Gosper's algorithm, which is applicable for a wide class of
   summands. Karr's theory of extension difference fields and some
```

heuristic techniques are touched on briefly."

```
@article{Abra85,
   author = "Abramov, S.A.",
   title = "Separation of variables in rational functions",
   year = "1985",
   journal = "USSR Computational Mathematics and Mathematical Physics",
   volume = "25",
   number = "5",
   pages = "99--102",
   paper = "Abra85.pdf",
   abstract = "
The problem of expanding a rational function of several variables into
```

terms with separable variables is formulated. An algorithm for solving this problem is given. Programs which implement this algorithm can occur in sets of algebraic alphabetical transformations on a computer and can be used to reduce the multiplicity of sums and integrals of rational functions for investigating differential equations with rational right-hand sides etc."

— axiom.bib —

```
@Article{Karr85,
  author = "Karr, Michael",
  title = "Theory of Summation in Finite Terms",
  year = "1985",
  journal = "Journal of Symbolic Computation",
  volume = "1",
  number = "3",
 month = "September",
 pages = "303-315",
 paper = "Karr85.pdf",
  abstract = "
   This paper discusses some of the mathematical aspects of an algorithm
    for finding formulas for finite sums. The results presented here
    concern a property of difference fields which show that the algorithm
    does not divide by zero, and an analogue to Liouville's theorem on
    elementary integrals."
}
```

```
@book{Koep98,
   author = "Koepf, Wolfram",
   title = "Hypergeometric Summation",
   publisher = "Springer",
   year = "1998",
   isbn = "978-1-4471-6464-7",
   paper = "Koep98.pdf",
   abstract = "
    Modern algorithmic techniques for summation, most of which were
   introduced in the 1990s, are developed here and carefully implemented
   in the computer algebra system Maple.
```

The algorithms of Fasenmyer, Gosper, Zeilberger, Petkovsek and van Hoeij for hypergeometric summation and recurrence equations, efficient multivariate summation as well as q-analogues of the above algorithms are covered. Similar algorithms concerning differential equations are considered. An equivalent theory of hyperexponential integration due to Almkvist and Zeilberger completes the book.

The combination of these results gives orthogonal polynomials and (hypergeometric and q-hypergeometric) special functions a solid algorithmic foundation. Hence, many examples from this very active field are given.

The materials covered are sutiable for an introductory course on algorithmic summation and will appeal to students and researchers alike."

— axiom.bib —

@InProceedings{Schn00, author = "Schneider, Carsten",

title = "An implementation of Karr's summation algorithm in Mathematica", year = "2000",

booktitle = "S\'eminaire Lotharingien de Combinatoire",

volume = "S43b", pages = "1-10",

url = "",
paper = "Schn00.pdf",

abstract = "

Implementations of the celebrated Gosper algorithm (1978) for indefinite summation are available on almost any computer algebra platform. We report here about an implementation of an algorithm by Karr, the most general indefinite summation algorithm known. Karr's algorithm is, in a sense, the summation counterpart of Risch's algorithm for indefinite integration. This is the first implementation of this algorithm in a major computer algebra system. Our version contains new extensions to handle also definite summation problems. In addition we provide a feature to find automatically appropriate difference field extensions in which a closed form for the summation problem exists. These new aspects are illustrated by a variety of examples."

}

```
@phdthesis{Schn01,
   author = "Schneider, Carsten",
   title = "Symbolic Summation in Difference Fields",
   school = "RISC Research Institute for Symbolic Computation",
   year = "2001",
   url =
        "http://www.risc.jku.at/publications/download/risc_3017/SymbSumTHESIS.pdf",
   paper = "Schn01.pdf",
   abstract = "
```

There are implementations of the celebrated Gosper algorithm (1978) on almost any computer algebra platform. Within my PhD thesis work I implemented Karr's Summation Algorithm (1981) based on difference field theory in the Mathematica system. Karr's algorithm is, in a sense, the summation counterpart of Risch's algorithm for indefinite integration. Besides Karr's algorithm which allows us to find closed forms for a big clas of multisums, we developed new extensions to handle also definite summation problems. More precisely we are able to apply creative telescoping in a very general difference field setting and are capable of solving linear recurrences in its context.

Besides this we find significant new insights in symbolic summation by rephrasing the summation problems in the general difference field setting. In particular, we designed algorithms for finding appropriate difference field extensions to solve problems in symbolic summation. For instance we deal with the problem to find all nested sum extensions which provide us with additional solutions for a given linear recurrence of any order. Furthermore we find appropriate sum extensions, if they exist, to simplify nested sums to simpler nested sum expressions. Moreover we are able to interpret creative telescoping as a special case of sum extensions in an indefinite summation problem. In particular we are able to determine sum extensions, in case of existence, to reduce the order of a recurrence for a definite summation problem."

}

— axiom.bib —

@inproceedings{Gerh03,

```
author = "Gerhard, J. and Giesbrecht, M. and Storjohann, A. and Zima, E.V.", title = "Shiftless decomposition and polynomial-time rational summation", booktitle = "Proceedings of ISSAC'03",
```

number = "17",

```
year = "2003",
  pages = "119--126",
  paper = "Gerh03.pdf",
  abstract = "
    New algorithms are presented for computing the dispersion set of two
   polynomials over \{\bf\ Q\} and for \{\sl\ shiftless\} factorization. Together
    with a summability criterion by Abramov, these are applied to get a
   polynomial-time algorithm for indefinite rational summation, using a
    sparse representation of the output."
}
            — axiom.bib —
@article{Schn05,
 author = "Schneider, Carsten",
  title = "A new Sigma approach to multi-summation",
  year = "2005",
  journal = "Advances in Applied Mathematics",
  volume = "34",
  number = "4",
  pages = "740--767",
  paper = "Schn05.pdf",
  abstract = "
    We present a general algorithmic framework that allows not only to
    deal with summation problems over summands being rational expressions
    in indefinite nested syms and products (Karr, 1981), but also over
    $\delta$-finite and holonomic summand expressions that are given by a
    linear recurrence. This approach implies new computer algebra tools
    implemented in Sigma to solve multi-summation problems efficiently.
    For instacne, the extended Sigma package has been applied successively
    to provide a computer-assisted proof of Stembridge's TSPP Theorem."
}
            — axiom.bib —
@article{Kaue08,
  author = "Kauers, Manuel and Schneider, Carsten",
  title = "Indefinite summation with unspecified summands",
  year = "2006",
  journal = "Discrete Mathematics",
 volume = "306",
```

volume = "56",

```
pages = "2073--2083",
  paper = "Kaue80.pdf",
  abstract = "
    We provide a new algorithm for indefinite nested summation which is
    applicable to summands involving unspecified sequences x(n). More
    than that, we show how to extend Karr's algorithm to a general
    summation framework by which additional types of summand expressions
    can be handled. Our treatment of unspecified sequences can be seen as
    a first illustrative application of this approach."
}
            — axiom.bib —
@article{Kaue07,
  author = "Kauers, Manuel",
  title = "Summation algorithms for Stirling number identities",
  year = "2007",
  journal = "Journal of Symbolic Computation",
  volume = "42",
 number = "10",
  month = "October",
  pages = "948--970",
  paper = "Kaue07.pdf",
  abstract = "
    We consider a class of sequences defined by triangular recurrence
    equations. This class contains Stirling numbers and Eulerian numbers
    of both kinds, and hypergeometric multiples of those. We give a
    sufficient criterion for sums over such sequences to obey a recurrence
    equation, and present algorithms for computing such recurrence
    equations efficiently. Our algorithms can be used for verifying many
    known summation identities on Stirling numbers instantly, and also for
    discovering new identities."
}
            — axiom.bib —
@InProceedings{Schn07,
  author = "Schneider, Carsten",
  title = "Symbolic Summation Assists Combinatorics",
  vear = "2007",
 booktitle = "S\'eminaire Lotharingien de Combinatoire",
```

```
article = "B56b",
  url = "",
  paper = "Schn07.pdf",
  abstract = "
    We present symbolic summation tools in the context of difference
    fields that help scientists in practical problem solving. Throughout
    this article we present multi-sum examples which are related to
    combinatorial problems."
}
            — axiom.bib —
@article{Schn08,
  author = "Schneider, Carsten",
  title = "A refined difference field theory for symbolic summation",
  year = "2008",
  journal = "Journal of Symbolic Computation",
  volume = "43",
  number = "9",
  pages = "611--644",
  paper = "Schn08.pdf",
  abstract = "
    In this article we present a refined summation theory based on Karr's
    difference field approach. The resulting algorithms find sum
    representations with optimal nested depth. For instance, the
    algorithms have been applied successively to evaluate Feynman
    integrals from Perturbative Quantum Field Theory"
}
            — axiom.bib —
@article{Schn09,
  author = "Schneider, Carsten",
  title = "Structural theorems for symbolic summation",
  journal = "Proc. AAECC-2010",
  year = "2010",
  volume = "21",
  pages = "1--32",
  paper = "Schn09.pdf",
  abstract = "
   Starting with Karr's structural theorem for summation - the discrete
    version of Liouville's structural theorem for integration - we work
```

out crucial properties of the underlying difference fields. This leads to new and constructive structural theorems for symbolic summation. E.g., these results can be applied for harmonic sums which arise frequently in particle physics."

— axiom.bib —

@article{Eroc10,
 author = {Er\"ocal, Bur\c{c}in},
 title = "Summation in Finite Terms Using Sage",
 journal = "ACM Commun. Comput. Algebra",
 volume = "44",
 number = "3/4",
 month = "January",
 year = "2011",
 issn = "1932-2240",
 pages = "190--193",
 url = "http://doi.acm.org/10.1145/1940475.1940517",
 publisher = "ACM",
 paper = "Eroc10.pdf",
 abstract = "

The summation analogue of the Risch integration algorithm developed by Karr uses towers of difference fields to model nested indefinite sums and products, as the Risch algorithm uses towers of differential fields to model the so called {\sl elementary functions}. The algorithmic machinery developed by Karr, and later generalized and extended, allows one to find solutions of first order difference equations over such towers of difference fields, in turn simplifying expressions involving sums and products.

We present an implementation of this machinery in the open source computer algebra system Sage. Due to the nature of open source software, this allows direct experimentation with the algorithms and structures involved while taking advantage of the state of the art primitives provided by Sage. Even though these methods are used behind the scenes in the summation package Sigma and they were previously implemented, this is the first open source implementation."

— axiom.bib —

}

```
@phdthesis{Eroc11,
   author = {Er\"ocal, Bur\c{c}in},
   title = "Algebraic Extensions for Symbolic Summation",
   school = "RISC Research Institute for Symbolic Computation",
   year = "2011",
   url =
        "http://www.risc.jku.at/publications/download/risc_4320/erocal_thesis.pdf",
   paper = "Eroc11.pdf",
   abstract = "
```

The main result of this thesis is an effective method to extend Karr's symbolic summation framework to algebraic extensions. These arise, for example, when working with expressions involving \$(-1)^n\$. An implementation of this method, including a modernised version of Karr's algorithm is presented.

Karr's algorithm is the summation analogue of the Risch algorithm for indefinite integration. In the summation case, towers of specialized difference fields called \$\prod\sum\-fields are used to model nested sums and products. This is similar to the way elementary functions involving nested logarithms and exponentials are represented in differential fields in the integration case.

In contrast to the integration framework, only transcendental extensions are allowed in Karr's construction. Algebraic extensions of \$\prod\sum\frac{1}{2} = 1 \text{ fields can even be rings with zero divisors. Karr's methods rely heavily on the ability to solve first-order linear difference equations and they are no longer applicable over these rings.

Based on Bronstein's formulation of a method used by Singer for the solution of differential equations over algebraic extensions, we transform a first-order linear equation over an algebraic extension to a system of first-order equations over a purely transcendental extension field. However, this domain is not necessarily a \$\prod\sum\-field. Using a structure theorem by Singer and van der Put, we reduce this system to a single first-order equation over a \$\prod\sum\-field, which can be solved by Karr's algorithm. We also describe how to construct towers of difference ring extensions on an algebraic extension, where the same reduction methods can be used.

A common bottleneck for symbolic summation algorithms is the computation of nullspaces of matrices over rational function fields. We present a fast algorithm for matrices over $\hat{Q}(x)$ which uses fast arithmetic at the hardware level with calls to BLAS subroutines after modular reduction. This part is joint work with Arne Storjohann."

}

```
@article{Poly11,
  author = "Polyadov, S.P.",
  title = "Indefinite summation of rational functions with factorization
           of denominators",
  year = "2011",
 month = "November",
  journal = "Programming and Computer Software",
  volume = "37",
  number = "6",
  pages = "322--325",
  paper = "Poly11.pdf",
  abstract = "
   A computer algebra algorithm for indefinite summation of rational
   functions based on complete factorization of denominators is
   proposed. For a given $f$, the algorithm finds two rational functions
   s, r such that f=g(x+1)-g(x)+r and the degree of the denominator
   of $r$ is minimal. A modification of the algorithm is also proposed
   that additionally minimizes the degree of the denominator of
   $g$. Computational complexity of the algorithms without regard to
   denominator factorization is shown to be 0(m^2), where m is the
   degree of the denominator of $f$."
}
```

```
@article{Schn13,
  author = "Schneider, Carsten",
  title =
  "Fast Algorithms for Refined Parameterized Telescoping in Difference Fields",
  journal = "CoRR",
  year = "2013",
  volume = "abs/1307.7887",
  paper = "Schn13.pdf",
  keywords = "survey",
  abstract = "
   Parameterized telescoping (including telescoping and creative
    telescoping) and refined versions of it play a central role in the
    research area of symbolic summation. In 1981 Karr introduced
    $\prod\sum$-fields, a general class of difference fields, that enables
    one to consider this problem for indefinite nested sums and products
    covering as special cases, e.g., the (q-)hypergeometric case and their
```

mixed versions. This survey article presents the available algorithms in the framework of $\scriptstyle \$ extensions and elaborates new results concerning efficiency."

— axiom.bib —

```
@article{Zima13,
   author = "Zima, Eugene V.",
   title = "Accelerating Indefinite Summation: Simple Classes of Summands",
   journal = "Mathematics in Computer Science",
   year = "2013",
   month = "December",
   volume = "7",
   number = "4",
   pages = "455--472",
   paper = "Zima13.pdf",
   abstract = "
```

We present the history of indefinite summation starting with classics (Newton, Montmort, Taylor, Stirling, Euler, Boole, Jordan) followed by modern classics (Abramov, Gosper, Karr) to the current implementation in computer algebra system Maple. Along with historical presentation we describe several ''acceleration techniques'' of algorithms for indefinite summation which offer not only theoretical but also practical improvements in running time. Implementations of these algorithms in Maple are compared to standard Maple summation tools"

— axiom.bib —

```
@misc{Schn14,
  author = "Schneider, Carsten",
  title = "A Difference Ring Theory for Symbolic Summation",
  year = "2014",
  paper = "Schn14.pdf",
  abstract = "
```

A summation framework is developed that enhances Karr's difference field approach. It covers not only indefinite nested sums and products in terms of transcendental extensions, but it can treat, e.g., nested products defined over roots of unity. The theory of the so-called \$R\prod\sum*\$-extensions is supplemented by algorithms that support the construction of such difference rings automatically and that assist in

the task to tackle symbolic summation problems. Algorithms are presented that solve parameterized telescoping equations, and more generally parameterized first-order difference equations, in the given difference ring. As a consequence, one obtains algorithms for the summation paradigms of telescoping and Zeilberger's creative telescoping. With this difference ring theory one obtains a rigorous summation machinery that has been applied to numerous challenging problems coming, e.g., from combinatorics and particle physics."

— axiom.bib —

@phdthesis{Vazq14,
 author = "Vazquez-Trejo, Javier",
 title = "Symbolic Summation in Difference Fields",
 year = "2014",
 school = "Carnegie-Mellon University",
 paper = "Vazq14.pdf",
 abstract = "

We seek to understand a general method for finding a closed form for a given sum that acts as its antidifference in the same way that an integral has an antiderivative. Once an antidifference is found, then given the limits of the sum, it suffices to evaluate the antidifference at the given limits. Several algorithms (by Karr and Schneider) exist to find antidifferences, but the apers describing these algorithms leave out several of the key proofs needed to implement the algorithms. We attempt to fill in these gaps and find that many of the steps to solve difference equations rely on being able to solve two problems: the equivalence problem and the homogenous group membership problem. Solving these two problems is essential to finding the polynomial degree bounds and denominator bounds for solutions of difference equations. We study Karr and Schneider's treatment of these problems and elaborate on the unproven parts of their work. Section 1 provides background material; section 2 provides motivation and previous work; Section 3 provides an outline of Karr's Algorithm; section 4 examines the Equivalance Problem, and section 5 examines the Homogeneous Group Membership Problem. Section 6 presents some proofs for the denominator and polynomial bounds used in solving difference equations, and Section 7 gives some directions for future work."

}

}

— axiom.bib — @book{Petk97, author = "Petkov\v{s}ek, Marko and Wilf, Herbert S. and Zeilberger, Doran", title = "A=B", publisher = "A.K. Peters, Ltd", year = "1997",paper = "Petk97.pdf" — axiom.bib — @misc{Temm14, author = "Temme, N.M.", title = "Bernoulli Polynomials Old and New", paper = "Temm14.pdf", abstract = " We consider two problems on generalized Bernoulli polynomials $B_n^u(z)$. One is connected with defining functions instead of polynomials by making the degree \$n\$ of the polynomial a complex variable. In the second problem we are concerned with the asymptotic behaviour of \$B_n^u(z)\$ when the degree \$n\$ tends to infinity." }

2.33 Differential Forms

— axiom.bib —

@book{Cart06,
 author = {Cartan, Henri},
 title = {Differential Forms},
 year = "2006",
 location = {Mineola, N.Y},
 edition = {Auflage: Tra},
 isbn = {9780486450100},
 pagetotal = {166},
 publisher = {Dover Pubn Inc},

```
date = \{2006-05-26\}
            — axiom.bib —
@book{Flan03,
 author = "Flanders, Harley",
 title = "Differential Forms with Applications to the Physical Sciences",
 year = "2003",
 location = "Mineola, N.Y",
  isbn = "9780486661698",
 pagetotal = "240",
 publisher = "Dover Pubn Inc",
 date = "2003-03-28",
  comment = "documentation for DeRhamComplex"
}
            — axiom.bib —
@book{Whit12,
  author = {Whitney, Hassler},
 title =
   {Geometric Integration Theory: Princeton Mathematical Series, No. 21},
 year = "2012",
  isbn = \{9781258346386\},
  shorttitle = {Geometric Integration Theory},
 pagetotal = \{402\},
 publisher = {Literary Licensing, {LLC}},
 date = \{2012-05-01\}
}
            — axiom.bib —
@book{Fede13,
 author = {Federer, Herbert},
 title = {Geometric Measure Theory},
```

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year = "2013",
        location = {Berlin ; New York},
        edition = {Reprint of the 1st ed. Berlin, Heidelberg, New York 1969},
        isbn = \{9783540606567\},
        pagetotal = {700},
        publisher = {Springer},
        date = \{2013-10-04\},
        abstract =
                 "This book is a major treatise in mathematics and is essential in the
                 working library of the modern analyst. (Bulletin of the London
                Mathematical Society)"
                                                  — axiom.bib —
@book{Abra93,
        author = "Abraham, Ralph and Marsden, Jerrold E. and Ratiu, Tudor",
        title = "Manifolds, Tensor Analysis, and Applications",
        year = "1993",
        location = "New York",
        edition = "2nd Corrected ed. 1988. Corr. 2nd printing 1993",
        isbn = "9780387967905",
        pagetotal = "656",
        publisher = "Springer",
        date = "1993-08-26",
        abstract = "
                The purpose of this book is to provide core material in nonlinear
                analysis for mathematicians, physicists, engineers, and mathematical % \left( 1\right) =\left( 1\right) \left( 1\right
                biologists. The main goal is to provide a working knowledge of
                manifolds, dynamical systems, tensors, and differential forms. Some
                 applications to Hamiltonian mechanics, fluid mechanics,
                 electromagnetism, plasma dynamics and control theory are given using
                 both invariant and index notation. The prerequisites required are
                solid undergraduate courses in linear algebra and advanced calculus."
}
                                                  — axiom.bib —
@book{Lamb97,
        author = {Lambe, L. A. and Radford, D. E.},
        title = {Introduction to the Quantum Yang-Baxter Equation and
                                              Quantum Groups: An Algebraic Approach},
```

```
year = "1997",
  location = {Dordrecht ; Boston},
  edition = {Auflage: 1997},
  isbn = \{9780792347217\},
  shorttitle = {Introduction to the Quantum Yang-Baxter Equation and
                Quantum Groups},
  abstract = {
    Chapter 1 The algebraic prerequisites for the book are covered here
    and in the appendix. This chapter should be used as reference material
    and should be consulted as needed. A systematic treatment of algebras,
    coalgebras, bialgebras, Hopf algebras, and represen tations of these
    objects to the extent needed for the book is given. The material here
    not specifically cited can be found for the most part in [Sweedler,
    1969] in one form or another, with a few exceptions. A great deal of
    emphasis is placed on the coalgebra which is the dual of n \times n
    matrices over a field. This is the most basic example of a coalgebra
    for our purposes and is at the heart of most algebraic constructions
    described in this book. We have found pointed bialgebras useful in
    connection with solving the quantum Yang-Baxter equation. For this
    reason we develop their theory in some detail. The class of examples
    described in Chapter 6 in connection with the quantum double consists
    of pointed Hopf algebras. We note the quantized enveloping algebras
    described Hopf algebras. Thus for many reasons pointed bialgebras are
    elsewhere are pointed of fundamental interest in the study of the
    quantum Yang-Baxter equation and objects quantum groups.},
  pagetotal = {300},
  publisher = {Springer},
  date = \{1997-10-31\}
            — axiom.bib —
@misc{Whee12,
  author = "Wheeler, James T.",
  title = "Differential Forms",
 year = "2012",
 month = "September",
"http://www.physics.usu.edu/Wheeler/ClassicalMechanics/CMDifferentialForms.pdf",
 paper = "Whee12.pdf"
```

2.34 To Be Classified

number = "85-12",

```
— axiom.bib —
@InProceedings{Kalt83,
 author = "Kaltofen, E.",
 title = "On the complexity of finding short vectors in integer lattices",
 booktitle = "Proc. EUROCAL '83",
 series = "Lect. Notes Comput. Sci.",
 year = "1983",
 volume = "162",
 pages = "236--244",
 publisher = "Springer-Verlag",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/83/Ka83_eurocal.pdf",
 paper = "Kalt83.pdf",
            — axiom.bib —
@InProceedings{Kalt85,
 author = "Kaltofen, E.",
 title = "Effective {Hilbert} Irreducibility",
 booktitle = "Proc. EUROSAM '84",
 pages = "275--284",
 crossref = "EUROSAM84",
 year = "1985",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_infcontr.ps.gz",
 paper = "Kalt85.ps",
            — axiom.bib —
@TechReport{Kalt85c,
 author = "Kaltofen, E.",
 title = "Sparse Hensel lifting",
 institution = "RPI",
 address = "Dept. Comput. Sci., Troy, N. Y.",
 year = "1985",
```

```
url = "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_techrep.pdf",
 paper = "Kalt85c.pdf",
            — axiom.bib —
@InProceedings{Kalt85d,
 author = "Kaltofen, E.",
 title = "Sparse Hensel lifting",
 booktitle = "EUROCAL 85 European Conf. Comput. Algebra Proc. Vol. 2",
  crossref = "EUROCAL85",
 pages = "4--17",
 year = "1985",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/85/Ka85_eurocal.pdf",
 paper = "Kalt85d.pdf",
}
            — axiom.bib —
@Article{Mill88,
  author = "Miller, G.L. and Ramachandran, V. and Kaltofen, E.",
 title = "Efficient parallel evaluation of straight-line code and
          arithmetic circuits",
  journal = "SIAM J. Comput.",
 year = "1988",
 volume = "17",
 number = "4",
 pages = "687--695",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/88/MRK88.pdf",
 paper = "Mill88.pdf",
            — axiom.bib —
@Article{Greg88,
 author = "Gregory, B.; Kaltofen, E.",
  title = "Analysis of the binary complexity of asymptotically fast
```

```
algorithms for linear system solving",
  journal = "SIGSAM Bulletin",
  year = "1988",
  month = "April",
  volume = "22",
  number = "2",
  pages = "41--49",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/88/GrKa88.pdf",
 paper = "Grey88.pdf",
}
            — axiom.bib —
@Article{Kalt89a,
  author = "Kaltofen, E.; Rolletschek, H.",
  title = "Computing greatest common divisors and factorizations in
           quadratic number fields",
  journal = "Math. Comput.",
  year = "1989",
  volume = "53",
  number = "188",
  pages = "697--720",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/89/KaRo89.pdf",
  paper = "Kalt89a.pdf",
            — axiom.bib —
@Unpublished{Kalt89b,
  author = "Kaltofen, E.",
  title = "Processor efficient parallel computation of polynomial greatest
           common divisors",
  year = "1989",
  month = "July",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/89/Ka89_gcd.ps.gz",
  paper = "Kalt89b.ps",
            — axiom.bib —
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@TechReport{Kalt89c,
  author = "Kaltofen, E.",
  title = "Parallel Algebraic Algorithm Design",
  institution = "RPI",
  address = "Dept. Comput. Sci., Troy, New York",
 year = "1989",
 month = "July",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/89/Ka89_parallel.ps.gz",
 paper = "Kalt89c.ps",
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@InProceedings{Cann89,
  author = "Canny, J. and Kaltofen, E. and Yagati, Lakshman",
 title = "Solving systems of non-linear polynomial equations faster",
 booktitle = "Proc. 1989 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC89",
 pages = "121--128",
 year = "1989",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/89/CKL89.pdf",
 paper = "Cann89.pdf",
            — axiom.bib —
@Article{Kalt90b,
 author = "Kaltofen, E.",
 title = "Computing the irreducible real factors and components of an
           algebraic curve",
  journal = "Applic. Algebra Engin. Commun. Comput.",
  year = "1990",
  volume = "1",
 number = "2",
 pages = "135--148",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/90/Ka90_aaecc.pdf",
 paper = "Kalt90b.pdf",
}
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@Article{Kalt90d,
 author = "Kaltofen, E.; Trager, B.",
 title = "Computing with polynomials given by black boxes for their
    evaluations: Greatest common divisors, factorization, separation of
   numerators and denominators",
  journal = "J. Symbolic Comput.",
  year = "1990",
  volume = "9",
 number = "3",
 pages = "301--320",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/90/KaTr90.pdf",
 paper = "Kalt90d.pdf",
            — axiom.bib —
@InProceedings{Kalt91a,
  author = "Kaltofen, E. and Singer, M.F.",
  editor = "D. V. Shirkov and V. A. Rostovtsev and V. P. Gerdt",
  title = "Size efficient parallel algebraic circuits for partial derivatives",
 booktitle =
   "IV International Conference on Computer Algebra in Physical Research",
  pages = "133--145",
  publisher = "World Scientific Publ. Co.",
  year = "1991",
  address = "Singapore",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/91/KaSi91.pdf",
 paper = "Kalt91a.pdf",
            — axiom.bib —
@InProceedings{Kalt93,
  author = "Kaltofen, E.",
  title = "Computational Differentiation and Algebraic Complexity Theory",
  booktitle = "Workshop Report on First Theory Institute on Computational
               Differentiation",
  editor = "C. H. Bischof and A. Griewank and P. M. Khademi",
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publisher = "Argonne National Laboratory",
  address = "Argonne, Illinois",
  series = "Tech. Rep.",
  volume = "ANL/MCS-TM-183",
 month = "December",
 year = "1993",
 pages = "28--30",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/93/Ka93_diff.pdf",
 paper = "Kalt93.pdf",
            — axiom.bib —
@Article{Kalt93b,
  author = "Kaltofen, E.",
 title = "Direct proof of a theorem by Kalkbrener, Sweedler, and Taylor",
  journal = "SIGSAM Bulletin",
 year = "1993",
 volume = "27",
 number = "4",
 pages = "2",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/93/Ka93_sambull.ps.gz",
 paper = "Kalt93b.ps",
            — axiom.bib —
@InProceedings{Kalt94,
  author = "Kaltofen, E. and Pan, V.",
 title = "Parallel solution of Toeplitz and Toeplitz-like linear
          systems over fields of small positive characteristic",
 booktitle = "Proc. First Internat. Symp. Parallel Symbolic Comput.",
  crossref = "PASCO94",
 pages = "225--233",
 year = "1994",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/94/KaPa94.pdf",
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}
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@InProceedings{Sama95,
  author = "Samadani, M. and Kaltofen, E.",
  title = "Prediction based task scheduling in distributed computing",
 booktitle = "Languages, Compilers and Run-Time Systems for Scalable
               Computers",
  editor = "B. K. Szymanski and B. Sinharoy",
  publisher = "Kluwer Academic Publ.",
  address = "Boston",
  pages = "317--320",
  year = "1996",
 url =
    "http://www.math.ncsu.edu/~kaltofen/bibliography/95/SaKa95_poster.ps.gz",
 paper = "Sama95.ps",
            — axiom.bib —
@InProceedings{Erli96,
 author = "Erlingsson, U. and Kaltofen, E. and Musser, D.",
  title = "Generic {Gram}-{Schmidt} Orthogonalization by Exact Division",
 booktitle = "Proc. 1996 Internat. Symp. Symbolic Algebraic Comput.",
  crossref = "ISSAC96",
  year = "1996",
 pages = "275--282",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/96/EKM96.pdf",
 paper = "Erli96.pdf",
            — axiom.bib —
@InProceedings{Kalt96,
author = "Kaltofen, E. and Lobo, A.",
title = "On rank properties of {Toeplitz} matrices over finite fields",
 booktitle = "Proc. 1996 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC96",
 year = "1996",
  pages = "241--249",
  url = "http://www.math.ncsu.edu/~kaltofen/bibliography/96/KaLo96_issac.pdf",
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paper = "Kalt96.pdf",
                                                              — axiom.bib —
@Article{Kalt97,
          author = "Kaltofen, E.",
          title = "Teaching Computational Abstract Algebra",
          journal = "Journal of Symbolic Computation",
          volume = "23",
          number = "5-6"
         pages = "503--515",
          year = "1997",
          note = "Special issue on education, L. Lambe, editor.",
          url = "http://www.math.ncsu.edu/~kaltofen/bibliography/97/Ka97_jsc.pdf",
          keywords = "axiomref,read",
         paper = "Kalt97.pdf",
          abstract = "
                    We report on the contents and pedagogy of a course in abstract algebra % \left( 1\right) =\left( 1\right) \left( 
                    that was taught with the aid of educational software developed within
                    the Mathematica system. We describe the topics covered and the
                    didactical use of the corresponding Mathematica packages, as well as
                    draw conclusions for future such courses from the students' comments
                    and our own experience."
}
                                                              — axiom.bib —
@Unpublished{Hitz97,
          author = "Hitz, M. A. and Kaltofen, E.",
          title = "The {Kharitonov} theorem and its applications in symbolic
                                                       mathematical computation",
          year = "1997",
          month = "May",
         url = "http://www.math.ncsu.edu/~kaltofen/bibliography/97/HiKa97_kharit.pdf",
         paper = "Hitz97.pdf",
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@InProceedings{Bern99,
  author = "Bernardin, L. and Char, B. and Kaltofen, E.",
  title = "Symbolic Computation in {Java}: an Appraisement",
 booktitle = "Proc. 1999 Internat. Symp. Symbolic Algebraic Comput.",
 crossref = "ISSAC99",
 year = "1999",
 pages = "237--244",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/99/BCK99.pdf",
 paper = "Bern99.pdf",
            — axiom.bib —
@InProceedings{Kalt02,
  author = "Kaltofen, Erich and McLean, Michael and Norris, Larry",
  title = "'{Using} {Maple} to Grade {Maple}' Assessment Software from
            {North Carolina State University}",
  booktitle = "Proceedings 2002 Maple Workshop",
  year = "2002",
 publisher = "Waterloo Maple Inc.",
 address = "Waterloo, Canada",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/02/KMN02.pdf",
 paper = "Kalt02.pdf",
            — axiom.bib —
@Book{Grab03,
  editor = "Grabmeier, J. and Kaltofen, E. and Weispfenning, V.",
  title = "Computer Algebra Handbook",
  publisher = "Springer-Verlag",
  year = "2003",
  note = "637 + xx~pages + CD-ROM. Includes E. Kaltofen and V. Weispfenning
    \S1.4 Computer algebra -- impact on research, pages 4--6;
    E. Kaltofen
    \S2.2.3 Absolute factorization of polynomials, page 26;
    E. Kaltofen and B. D. Saunders
    \S2.3.1 Linear systems, pages 36--38;
    R. M. Corless, E. Kaltofen and S. M. Watt
    \S2.12.3 Hybrid methods, pages 112--125;
    E. Kaltofen
    \S4.2.17 FoxBox and other blackbox systems, pages 383--385.",
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isbn = "3-540-65466-6",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/01/symnum.pdf",
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            — axiom.bib —
@InProceedings{Kalt07,
  author = "Kaltofen, Erich and Li, Bin and Sivaramakrishnan, Kartik and
           Yang, Zhengfeng and Zhi, Lihong",
  title = "Lower bounds for approximate factorizations via semidefinite
          programming (extended abstract)",
  year = "2007",
  booktitle =
    "SNC'07 Proc. 2007 Internat. Workshop on Symbolic-Numeric Comput.",
  crossref = "SNCO7",
  pages = "203--204",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/07/KLSYZ07.pdf",
 paper = "Kalt07.pdf",
            — axiom.bib —
@Article{Borw07,
  author = "Borwein, Peter and Kaltofen, Erich and Mossinghoff, Michael J.",
  title = "Irreducible Polynomials and {Barker} Sequences",
  journal = "{ACM} Communications in Computer Algebra",
  volume = "162",
 number = "4",
 year = "2007",
 pages = "118--121",
 month = "December",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/07/BKM07.pdf",
 paper = "Borw07.pdf",
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@Article{Kalt10,
  author = "Kaltofen, Erich and Lavin, Mark",
  title = "Efficiently Certifying Non-Integer Powers",
  journal = "Computational Complexity",
 year = "2010",
 volume = "19",
 number = "3",
 month = "September",
 pages = "355--366",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/09/KaLa09.pdf",
 paper = "Kalt10.pdf",
            — axiom.bib —
@InProceedings{Kalt11,
  author = "Kaltofen, Erich L. and Nehring, Michael",
  title = "Supersparse black box rational function interpolation",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'11",
 crossref = "ISSAC11",
 month = "June",
 year = "2011",
 pages = "177--185",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/11/KaNe11.pdf",
 paper = "Kalt11.pdf",
            — axiom.bib —
@InProceedings{Gren11a,
  author = "Grenet, Bruno and Kaltofen, Erich L. and Koiran, Pascal
           and Portier, Natacha",
  title = "Symmetric Determinantal Representation of Weakly Skew Circuits",
  booktitle = "Proc. 28th Internat. Symp. on Theoretical Aspects of Computer
              Science, STACS 2011",
  crossref = "STACS11",
 pages = "543--554",
 year = "2011",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/11/GKKP11.pdf",
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@InProceedings{Kalt11a,
  author = "Kaltofen, Erich L. and Nehring, Michael and Saunders, David B.",
  title = "Quadratic-Time Certificates in Linear Algebra",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'11",
  crossref = "ISSAC11",
 month = "June",
 year = "2011",
 pages = "171--176",
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 paper = "Kalt11a.pdf",
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@InProceedings{Kalt11b,
  author = "Kaltofen, Erich L. and Lee, Wen-shin and Yang, Zhengfeng",
  title = "Fast estimates of {Hankel} matrix condition numbers
           and numeric sparse interpolation",
  booktitle = "Proc. 2011 Internat. Workshop on Symbolic-Numeric Comput.",
 month = "June",
  crossref = "SNC11",
 year = "2011",
 pages = "130--136",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/11/KLY11.pdf",
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}
            — axiom.bib —
@InProceedings{Guo12,
  author = "Guo, Feng and Kaltofen, Erich L. and Zhi, Lihong",
  title = "Certificates of Impossibility of {Hilbert}-{Artin} Representations
          of a Given Degree for Definite Polynomials and Functions",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'12",
  crossref = "ISSAC12",
 month = "July",
  year = "2012",
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pages = "195--202",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/12/GKZ12.pdf",
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           — axiom.bib —
@InProceedings{Come12a,
 author = "Comer, Matthew T. and Kaltofen, Erich L. and Pernet, Cl{\'e}ment",
  title = "Sparse Polynomial Interpolation and {Berlekamp}/\allowbreak
           {Massey} Algorithms That Correct Outlier Errors in Input Values",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'12",
 crossref = "ISSAC12",
 month = "July",
 year = "2012",
 pages = "138--145",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/12/CKP12.pdf",
 paper = "Come12a.pdf",
           — axiom.bib —
@InProceedings{Boye13,
  author = "Boyer, Brice and Comer, Matthew T. and Kaltofen, Erich L.",
  title = "Sparse Polynomial Interpolation by Variable Shift in
           the Presence of Noise and Outliers in the Evaluations",
 booktitle =
   "Proc. Tenth Asian Symposium on Computer Mathematics (ASCM 2012)",
 year = "2013",
 month = "October",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/13/BCK13.pdf",
 paper = "Boye13.pdf",
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@InProceedings{Kalt13b,
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author = "Kaltofen, Erich and Yang, Zhengfeng",
  title = "Sparse multivariate function recovery from values with noise and
           outlier errors",
  year = "2013",
  booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'13",
  crossref = "ISSAC13",
 pages = "219--226",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/13/KaYa13.pdf",
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            — axiom.bib —
@InProceedings{Kalt13c,
  author = "Kaltofen, Erich L.",
 title = "Symbolic Computation and Complexity Theory Transcript of My Talk",
   "Proc. Tenth Asian Symposium on Computer Mathematics (ASCM 2012)",
 vear = "2013",
 month = "October",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/13/Ka13.pdf",
 paper = "Kalt13c.pdf",
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@InProceedings{Kalt14,
  author = "Kaltofen, Erich L. and Yang, Zhengfeng",
 title = "Sparse Multivariate Function Recovery With a High Error Rate
          in Evaluations",
 year = "2014",
  booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'14",
  crossref = "ISSAC14",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/14/KaYa14.pdf",
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@InProceedings{Kalt14a,
  author = "Kaltofen, Erich L. and Pernet, Cl{\'e}ment",
  title = "Sparse Polynomial Interpolation Codes and Their Decoding
          Beyond Half the Minimal Distance",
 year = "2014",
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 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/14/KaPe14.pdf",
 paper = "Kalt14a.pdf",
            — axiom.bib —
@InProceedings{Duma14,
  author = "Dumas, Jean-Guillaume and Kaltofen, Erich L.",
 title = "Essentially Optimal Interactive Certificates In Linear Algebra",
 year = "2014",
 booktitle = "Internat. Symp. Symbolic Algebraic Comput. ISSAC'14",
 crossref = "ISSAC14",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/14/DuKa14.pdf",
 paper = "Duma14.pdf",
            — axiom.bib —
@InProceedings{Boye14,
  author = "Boyer, Brice B. and Kaltofen, Erich L.",
  title = "Numerical Linear System Solving With Parametric Entries By
          Error Correction",
  year = "2014",
 booktitle = "SNC'14 Proc. 2014 Int. Workshop on Symbolic-Numeric Comput.",
 crossref = "SNC14",
 url = "http://www.math.ncsu.edu/~kaltofen/bibliography/14/BoKa14.pdf",
 paper = "Boye14.pdf",
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2.35 Axiom Citations in the Literature

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— ignore —
\bibitem[ACM 89]{ACM89} ACM, editor
Proceedings of the ACM-SIGSAM 1989 International
Symposium on Symbolic and Algebraic Computation, ISSAC '89 ACM Press,
New York, NY 10036, USA, 1989, , LCCN QA76.95.I59
 year = "1989",
  isbn = "0-89791-325-6",
 keywords = "axiomref",
            — ignore —
\bibitem[ACM 94]{ACM94} ACM, editor
ISSAC '94. Proceedings of the International
Symposium on Symbolic and Algebraic Computation. ACM Press, New York, NY,
10036, USA, 1994, . LCCN QA76.95.159
 year = "1994",
  isbn = "0-89791-638-7",
 keywords = "axiomref",
            — axiom.bib —
@article{Augo91,
  author = "Augot, D. and Charpin, P. and Sendrier, N.",
  title = "The miniumum distance of some binary codes via the
          Newton's identities",
  journal = "Cohen and Charping [CC91]",
  year = "1991",
  pages = "65-73",
  isbn = "0-387-54303-1",
 misc = "3-540-54303-1 (Berlin). LCCN QA268.E95 1990",
 keywords = "axiomref",
 paper = "Augo91.pdf",
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— ignore — \bibitem[Adams 94]{AL94} author = "Adams, William W. and Loustaunau, Philippe", title = "An Introduction to Gr\"obner Bases", year = "1994", American Mathematical Society (1994) isbn = "0-8218-3804-0",keywords = "axiomref", — ignore — \bibitem[Andrews 84]{And84} author = "Andrews, George E.", title = "Ramanujan and SCRATCHPAD", year = "1984",pages = "383-??", keywords = "axiomref", In Golden and Hussain [GH84] — ignore — \bibitem[Andrews 88]{And88} author = "Andrews, G. E.", title = "Application of Scratchpad to problems in special functions and combinatorics", year = "1988" pages = "158-??", isbn = "3-540-18928-9",keywords = "axiomref", In Janssen [Jan88], pages 158-?? ISBN 0-387-18928-9 LCCN QA155.7.E4T74

\bibitem[Anon 91]{Ano91}

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author = "Anonymous",
  year = "1991,
  keywords = "axiomref",
Proceedings 1991 Annual Conference, American Society for
Engineering Education. Challenges of a Changing World. ASEE, Washington, DC
2 vol.
            — ignore —
\bibitem[Anon 92]{Ano92}
  author = "Anonymous",
 year = "1992",
 keywords = "axiomref",
Programming environments for high-level scientific problem solving.
IFIP TC2/WG 2.5 working conference. IFIP Transactions. A Computer Science
and Technology, A-2:??, CODEN ITATEC. ISSN 0926-5473
            — ignore —
\bibitem[Anono 95]{Ano95}
  author =Anonymous
 keywords = "axiomref",
 year = "1995",
GAMM 94 annual meeting. Zeitschrift fur Angewandte Mathematik und
Physik, 75 (suppl. 2), CODEN ZAMMAX, ISSN 0044-2267
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@article{Bacl14,
   author = "Baclawski, Krystian",
   title = "SPAD language type checker",
   journal = "unknown",
   year = "2014",
   url = "http://github.com/cahirwpz/phd",
```

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keywords = "axiomref",
  abstract = "
   The project aims to deliver a new type checker for SPAD language.
   Several improvements over current type checker are planned.
   \begin{itemize}
   \item introduce better type inference
   \item introduce modern language constructs
   \item produce understandable diagnostic messages
   \item eliminate well known bugs in the type system
   \item find new type errors
   \end{itemize}"
            — ignore —
\bibitem[Blair 70]{BGJ70}
  author = "Blair, Fred W. and Griesmer, James H. and Jenks, Richard D.",
  title = "An interactive facility for symbolic mathematics",
  year = "1970",
  pages = "394-419",
  keywords = "axiomref",
Proc. International Computing Symposium, Bonn, Germany,
            — ignore —
\bibitem[Blair 70a]{BJ70}
  author = "Blair, Fred W. and Jenks, Richard D.",
  title = "LPL: LISP programming language",
  year = "1970",
  keywords = "axiomref",
IBM Research Report, RC3062 Sept
            — axiom.bib —
@inproceedings{BGDW95,
  author = "Broadbery, P. A. and G(\) mez-D(\) author = T. and Watt, S. M.",
  title = "On the Implementation of Dynamic Evaluation",
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year = "1995",
pages = "77-84",
keywords = "axiomref",
isbn = "0-89791-699-9"
url = "http://pdf.aminer.org/000/449/014/on_the_implementation_of_dynamic_evaluation.pdf",
paper = "BGDW95.pdf",
abstract = "
 Dynamic evaluation is a technique for producing multiple results
 according to a decision tree which evolves with program execution.
 Sometimes it is desired to produce results for all possible branches
 in the decision tree, while on other occasions, it may be sufficient
 to compute a single result which satisfies certain properties. This
 techinique finds use in computer algebra where computing the correct
 result depends on recognizing and properly handling special cases of
 parameters. In previous work, programs using dynamic evaluation have
 explored all branches of decision trees by repeating the computations
 prior to decision points.
 This paper presents two new implementations of dynamic evaluation
 which avoid recomputing intermediate results. The first approach uses
 Scheme ''continuations'' to record state for resuming program
 execution. The second implementation uses the Unix "fork" operation
 to form new processes to explore alternative branches in parallel."
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— axiom.bib —

}

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@inproceedings{Boe89,
  author = "Boehm, Hans-J.",
  title = "Type Inference in the Presence of Type Abstraction",
  year = "1989",
  pages = "192-206",
  keywords = "axiomref",
  url = "http://www.acm.org/pubs/citations/proceedings/pldi/73141/p192-boehm",
  paper = "Boe89.pdf",
  booktitle = "ACM SIGPLAN Notices",
  volume = "24",
  number = "7",
  month = "July"
  abstract = "
    A number of recent programming language designs incorporate a type
    checking system based on the Girard-Reynolds polymorphic
    $\lambda$-calculus. This allows the construction of general purpose,
    reusable software without sacrificing compile-time type checking. A
    major factor constraining the implementation of these languages is the
    difficulty of automatically inferring the lengthy type information
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}

that is otherwise required if full use is made of these languages. There is no known algorithm to solve any natural and fully general formulation of the ''type inference'' problem. One very reasonable formulation of the problem is known to be undecidable.

Here we define a restricted version of the type inference problem and present an efficient algorithm for its solution. We argue that the restriction is sufficiently weak to be unobtrusive in practice."

— axiom.bib —

@inproceedings{BHGMO4,

title = "Design verification for control engineering",

year = "2004",

month = "April",

booktitle = "Proc 4th Int. Conf. on Integrated Formal Methods",

keywords = "axiomref",

abstract = "

}

We introduce control engineering as a new domain of application for formal methods. We discuss design verification, drawing attention to the role played by diagrammatic evaluation criteria involving numeric plots of a design, such as Nichols and Bode plots. We show that symbolic computation and computational logic can be used to discharge these criteria and provide symbolic, automated, and very general alternatives to these standard numeric tests. We illustrate our work with reference to a standard reference model drawn from military avionics."

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\bibitem[Boulanger 91]{Bou91}
 author = "Boulanger, Jean-Louis",
 title = "Etude de la compilation de scratchpad 2",
 year = "1991",
 month = "September",
Rapport de DEA Universite dl lille 1
 keywords = "axiomref",

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— axiom.bib —
@misc{Bou93a,
  author = "Boulanger, Jean-Louis",
 title = "Axiom, language fonctionnel \'a d\'evelopement objet",
 year = "1993",
 month = "October",
 paper = "Bou93a.pdf",
 keywords = "axiomref"
            — axiom.bib —
@misc{Bou93b,
  author = "Boulanger, Jean-Louis",
  title = "AXIOM, A Functional Language with Object Oriented Development",
  year = "1993",
  paper = "Bou93b.pdf",
 keywords = "axiomref",
  abstract = "
   We present in this paper, a study about the computer algebra system
   Axiom, which gives us many very interesting Software engineering
   concepts. This language is a functional language with an Object
   Oriented Development. This feature is very important for modeling the
   mathematical world (Hierarchy) and provides a running with
   mathematical sense. (All objects are functions). We present many
   problems of running and development in Axiom. We can note that Aiom is
   the only system of this category."
}
            — ignore —
\bibitem[Boulanger 94]{Bou94}
  author = "Boulanger, J.L.",
  title = "Object Oriented Method for Axiom",
 year = "1995",
 month = "February",
 pages = "33-41",
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paper = "Bou94.pdf",

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ACM SIGPLAN Notices, 30(2) CODEN SINODQ ISSN 0362-1340
  keywords = "axiomref",
  abstract = "
    Axiom is a very powerful computer algebra system which combines two
    language paradigms (functional and OOP). Mathematical world is complex
    and mathematicians use abstraction to design it. This paper presents
    some aspects of the object oriented development in Axiom. The Axiom
    programming is based on several new tools for object oriented
    development, it uses two levels of class and some operations such that
    {\sl coerce}, {\sl retract}, or {\sl convert} which permit the type
    evolution. These notions introduce the concept of multi-view."
            — ignore —
\bibitem[Bronstein 87]{Bro87}
  author = "Bronstein, Manuel",
  title = "Integration of Algebraic and Mixed Functions",
  year = "1987",
in [Wit87], p18
  keywords = "axiomref",
            — ignore —
\bibitem[Bronstein 89]{Bro89}
  author= "Bronstein, M.",
  title = "Simplification of real elementary functions",
  year = "1989",
 pages = "207-211",
  isbn = "0-89791-325-6",
ACM [ACM89] pages LCCN QA76.95.I59 1989
  keywords = "axiomref",
  abstract = "
    We describe an algorithm, based on Risch's real structure theorem, that
    determines explicitly all the algebraic relations among a given set of
    real elementary functions. We also provide examples from its
    implementation that illustrate the advantages over the use of complex
    logarithms and exponentials."
}
```

— axiom.bib —

}

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@inproceedings{Bron91a,
  author = "Bronstein, M.",
  title = "The Risch Differential Equation on an Algebraic Curve",
  booktitle = "Proc. 1991 Int. Symp. on Symbolic and Algebraic Computation",
  series = "ISSAC'91",
  year = "1991",
  pages = "241-246",
  isbn = "0-89791-437-6",
  publisher = "ACM, NY",
  keywords = "axiomref",
  paper = "Bro91a.pdf",
  abstract = "
    We present a new rational algorithm for solving Risch differential
    equations over algebraic curves. This algorithm can also be used to
    solve n^{th}\- order linear ordinary differential equations with
    coefficients in an algebraic extension of the rational functions. In
    the general (''mixed function'') case, this algorithm finds the
    denominator of any solution of the equation."
}
            — ignore —
\bibitem[Bronstein 91c]{Bro91c}
  author = "Bronstein, Manuel",
  title = "Computer Algebra and Indefinite Integrals",
  year = "1991",
 paper = "Bro91c.pdf",
in Computer Aided Proofs in Analysis, K.R. Meyers et al. (eds)
Springer-Verlag, NY (1991)
  keywords = "axiomref",
  abstract = "
    We give an overview, from an analytical point of view, of decision
    procedures for determining whether an elementary function has an
    elementary function has an elementary antiderivative. We give examples
    of algebraic functions which are integrable and non-integrable in
    closed form, and mention the current implementation of various computer
    algebra systems."
```

\bibitem[Bronstein 92]{Bro92} author = "Bronstein, M.", title = "Linear Ordinary Differential Equations: Breaking Through the Order 2 Barrier", year = "1992",url = "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/issac92.ps.gz", paper = "Bro92.pdf", keywords = "axiomref", abstract = " A major subproblem for algorithms that either factor ordinary linear differential equations or compute their closed form solutions is to find their solutions y which satisfy $y^{'}/y \in \operatorname{K}(x)$ where \$K\$ is the constant field for the coefficients of the equation. While a decision procedure for this subproblem was known in the \$19^{th}\$ century, it requires factoring polynomials over \$\overline{K}\$ and has not been implemented in full generality. We present here an efficient algorithm for this subproblem, which has been implemented in the AXIOM computer algebra system for equations of arbitrary order over arbitrary fields of characteristic O. This algorithm never needs to compute with the individual complex singularities of the equation, and algebraic numbers are added only when they appear in the potential solutions. Implementation of the complete Singer algorithm for n=2,3 based on this building block is in progress." }

— ignore —

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\bibitem[Bronstein 93]{Bro93}
  author = "Bronstein, Manuel (ed)",
  year = "1993",
  month = "July"
  isbn = "0-89791-604-2",
ISSAC'93: proceedings of the 1993 International Symposium on Symbolic
and Algebraic Computation, Kiev, Ukraine,
ACM Press New York, NY 10036, USA, ISBN
LCCN QA76.95 I59 1993 ACM order number 505930
  keywords = "axiomref",
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\bibitem[Brunelli 08]{Brun08}
author = "Brunelli, J.C.",
title = "Streams and Lazy Evaluation Applied to Integrable Models",
year = "2008",
url = "http://arxiv.org/PS_cache/nlin/pdf/0408/0408058v1.pdf",
paper = "Brun08.pdf",
keywords = "axiomref",
abstract = "
   Computer algebra procedures to manipulate pseudo-differential
   operators are implemented to perform calculations with integrable
   models. We use lazy evaluation and streams to represent and operate
   with pseudo-differential operators. No order of truncation is needed
   since terms are produced on demand. We give a series of concrete
   examples using the computer algebra language MAPLE."
```

— ignore —

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\bibitem[Bronstein 93]{BS93}
  author = "Bronstein, Manuel and Salvy, Bruno",
  title = "Full Partial Fraction Decomposition of Rational Functions",
  year = "1993",
  pages = "157-160",
  isbn = "0-89791-604-2",
In Bronstein [Bro93]   LCCN QA76.95 I59 1993
  keywords = "axiomref",
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— axiom.bib —

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@misc{Bro92a,
   author = "Bronstein, Manuel",
   title = "Integration and Differential Equations in Computer Algebra",
   year = "1992",
   url = "http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.42.576",
   paper = "Bro92a.pdf",
   keywords = "axiomref",
   abstract = "
   We describe in this paper how the problems of computing indefinite
   integrals and solving linear ordinary differential equations in closed
```

}

form are now solved by computer algebra systems. After a brief review of the mathematical history of those problems, we outline the two major algorithms for them (respectively the Risch and Singer algorithms) and the recent improvements on those algorithms which has allowed them to be implemented."

— ignore —

— ignore —

```
\bibitem[Bronstein 97a]{Bro97a}
 author = "Bronstein, Manuel and Weil, Jacques-Arthur",
 title = "On Symmetric Powers of Differential Operators",
 series = "ISSAC'97",
 year = "1997",
 pages = "156-163",
 keywords = "axiomref",
 url =
    "http://www-sop.inria.fr/cafe/Manuel.Bronstein/publications/mb_papers.html"
 paper = "Bro97a.pdf",
 publisher = "ACM, NY",
 abstract = "
   We present alternative algorithms for computing symmetric powers of
   linear ordinary differential operators. Our algorithms are applicable
   to operators with coefficients in arbitrary integral domains and
   become faster than the traditional methods for symmetric powers of
    sufficiently large order, or over sufficiently complicated coefficient
    domains. The basic ideas are also applicable to other computations
    involving cyclic vector techniques, such as exterior powers of
   differential or difference operators."
```

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— ignore —
\bibitem[Borwein 00]{Bor00}
  author = "Borwein, Jonathan",
 title = "Multimedia tools for communicating mathematics",
 year = "2000",
 pages = "58",
 isbn = "3-540-42450-4",
 publisher = "Springer-Verlag",
 keywords = "axiomref"
            — axiom.bib —
@article{BT94,
  author = "Brown, R. and Tonks, A.",
 title = "Calculations with simplicial and cubical groups in AXIOM",
  journal = "Journal of Symbolic Computation",
  volume = "17",
 number = "2",
 pages = "159-179",
 year = "1994",
 month = "February",
 misc = "CODEN JSYCEH ISSN 0747-7171",
 keywords = "axiomref"
            — axiom.bib —
@misc{Brow95,
  author = "Brown, Ronald and Dreckmann, Winfried",
 title = "Domains of data and domains of terms in AXIOM",
  year = "1995",
  keywords = "axiomref",
  paper = "DB95.pdf",
   The main new concept we wish to illustrate in this paper is a
```

distinction between ''domains of data'' and ''domains of terms'', and

its use in the programming of certain mathematical structures. Although this distinction is implicit in much of the programming work

```
that has gone into the construction of Axiom categories and domains,
    we believe that a formalisation of this is new, that standards and
    conventions are necessary and will be useful in various other
    contexts. We shall show how this concept may be used for the coding of
    free categories and groupoids on directed graphs."
}
            — ignore —
\bibitem[Buchberger 85]{BC85} Buchberger, Bruno and Caviness, Bob F. (eds)
EUROCAL '85: European Conference on Computer Algebra, Linz, Austria,
LLCN QA155.7.E4 E86
  isbn = "0-387-15983-5, 0-387-15984-3",
  year = "1985",
  month = "April",
  publisher = "Springer-Verlag, Berlin, Germany",
  keywords = "axiomref",
  misc = "Lecture Notes in Computer Science, Vol 204",
            — axiom.bib —
@misc{Buh05,
  author = "Buhl, Soren L.",
  title = "Some Reflections on Integrating a Computer Algebra System in R",
  year = "2005",
 keywords = "axiomref"
            — ignore —
\bibitem[Burge 91]{Burg91}
  author = "Burge, W.H.",
  title = "Scratchpad and the Rogers-Ramanujan identities",
  year = "1991",
  pages = "189-190",
  isbn = "0-89791-437-6",
  keywords = "axiomref",
```

```
abstract = "
   This note sketches the part played by Scratchpad in obtaining new
   proofs of Euler's theorem and the Rogers-Ramanujan Identities."
            — axiom.bib —
@techreport{BW87,
  author = "Burge, W. and Watt, S.",
  title = "Infinite structures in SCRATCHPAD II",
 year = "1987",
  institution = "IBM Research",
 type = "Technical Report",
 number = "RC 12794",
 keywords = "axiomref"
           — ignore —
\bibitem[Burge 87a]{BWM87}
  author = "Burge, William H. and Watt, Stephen M. and Morrison, Scott C.",
 title = "Streams and Power Series",
 year = "1987",
 pages = "9-12",
 keywords = "axiomref",
in [Wit87], pp9-12
            — ignore —
\bibitem[Burge 89]{BW89}
  author = "Burge, W. H. and Watt, S. M.",
  title = "Infinite structures in Scratchpad II",
 year = "1989",
 pages = "138-148",
 isbn = "3-540-51517-8",
 keywords = "axiomref",
in Davenport [Dav89], LCCN QA155.7.E4E86 1987
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- ignore -
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\bibitem[Calmet 94]{Cal94} Calmet, J. (ed)
Rhine Workshop on Computer Algebra, Proceedings.
Universit{\"a}t Karsruhe, Karlsruhe, Germany 1994
 keywords = "axiomref",

- ignore -

\bibitem[Camion 92]{CCM92}

— ignore —

\bibitem[Capriotti 00]{CCR00}

author = "Capriotti, Olga and Cohen, Arjeh M. and Riem, Manfred",
title = "Java Phrasebooks for Computer Algebra and Automated Deduction",
url = "http://www.sigsam.org/bulletin/articles/132/paper8.pdf",
paper = "CCR00.pdf",
keywords = "axiomref",

— axiom.bib —

@misc{CC99,

author = "Capriotti, O. and Carlisle, D.",
title = "OpenMath and MathML: Semantic Mark Up for Mathematics",
year = "1999",

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url = "http://www.acm.org/crossroads/xrds6-2/openmath.html",
 keywords = "axiomref"
            — axiom.bib —
@misc{Capr99,
  author = "Capriotti, Olga and Cohen, Arjeh M. and Cuypers, Hans and
           Sterk, Hans",
 title = "OpenMath Technology for Interactive Mathematical Documents",
 vear = "2002",
 pages = "51-66",
 publisher = "Springer-Verlag, Berlin, Germany",
 url = "http://www.win.tue.nl/~hansc/lisbon.pdf",
 paper = "Capr99.pdf",
 misc = "in Multimedia Tools for Communicating Mathematics",
 keywords = "axiomref"
            — axiom.bib —
@misc{Carp04,
  author = "Carpent, Quentin and Conil, Christophe",
  title = "Utilisation de logiciels libres pour la r\'ealisation de TP MT26",
 year = "2004",
 paper = "Carp04.pdf",
 keywords = "axiomref"
           - axiom.bib -
@misc{Chu85,
  author = "Chudnovsky, D.V and Chudnovsky, G.V.",
 title = "Elliptic Curve Calculations in Scratchpad II",
 year = "1985",
  institution = "Mathematics Dept., IBM Research",
  type = "Scratchpad II Newsletter 1 (1)",
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keywords = "axiomref"
           - ignore -
\bibitem[Chudnovsky 87]{Chu87}
  author = "Chudnovsky, D.V and Chudnovsky, G.V.",
  title = "New Analytic Methods of Polynomial Root Finding",
 year = "1987",
 pages = "2",
 keywords = "axiomref",
in [Wit87]
            - ignore -
\bibitem[Chudnovsky 89]{Chu89}
  author = "Chudnovsky, D.V. and Chudnovsky, G.V.",
  title = "The computation of classical constants",
  year = "1989",
  month = "November",
  pages = "8178-8182",
  keywords = "axiomref",
Proc. Natl. Acad. Sci. USA Vol 86
           — axiom.bib —
@proceedings{CJ86,
  editor = "Chudnovsky, David and Jenks, Richard",
  title = "Computers in Mathematics",
  year = "1986",
 month = "July",
 isbn = "0-8247-8341-7",
 note = "International Conference on Computers and Mathematics",
  publisher = "Marcel Dekker, Inc",
  keywords = "axiomref"
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title = "Interactive Mathematical Documents on the Web",
year = "2003",
pages = "289-306",
editor = "Joswig, M. and Takayma, N.",
publisher = "Springer-Verlag, Berlin, Germany",
keywords = "axiomref",
misc = "in Algebra, Geometry and Software Systems"

— ignore —

\bibitem[Cohen 91]{CC91} Cohen, G.; Charpin, P.; (ed)
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/ Heidelberg, Germany / London, UK / etc., 1991 ISBN 0-387-54303-1
(New York), 3-540-54303-1 (Berlin), LCCN QA268.E95 1990
keywords = "axiomref",

— ignore —

\bibitem[Conrad (a)]{CFMPxxa}

paper = "CFMPxxa.pdf",

abstract = "

It is well-known that few object-oriented programming languages allow objects to change their nature at run-time. There have been a number of reasons presented for this, but it appears that there is a real need for matters to change. In this paper we discuss the need for object-oriented programming languages to reflect the dynamic nature of problems, particularly those arising in a mathematical context. It is from this context that we present a framework that realistically

@inproceedings{Dalm97,

represents the dynamic and evolving characteristic of problems and algorithms." — axiom.bib — @misc{CFMPxxb, author = "Conrad, Marc and French, Tim and Maple, Carsten and Pott, Sandra", title = "Mathematical Use Cases lead naturally to non-standard Inheritance Relationships: How to make them accessible in a mainstream language?", paper = "CFMPxxb.pdf", keywords = "axiomref", abstract = " Conceptually there is a strong correspondence between Mathematical Reasoning and Object-Oriented techniques. We investigate how the ideas of Method Renaming, Dynamic Inheritance and Interclassing can be used to strengthen this relationship. A discussion is initiated concerning the feasibility of each of these features." } — axiom.bib — @misc{Cuyp10, author = "Cuypers, Hans and Hendriks, Maxim and Knopper, Jan Willem", title = "Interactive Geometry inside MathDox", year = "2010",url = "http://www.win.tue.nl/~hansc/MathDox_and_InterGeo_paper.pdf", paper = "Cuyp10", keywords = "axiomref" } \mathbf{D} — axiom.bib —

author = {Dalmas, St\'ephane and Ga\"etano, Marc and Watt, Stephen},

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title = "An OpenMath 1.0 Implementation",
  booktitle = "Proc. 1997 Int. Symp. on Symbolic and Algebraic Computation",
  series = "ISSAC'97",
  year = "1997",
 isbn = "0-89791-875-4",
  location = "Kihei, Maui, Hawaii, USA",
  pages = "241-248",
  numpages = "8",
  url = "http://doi.acm.org/10.1145/258726.258794",
 doi = "10.1145/258726.258794",
  acmid = "258794",
 publisher = "ACM, New York, NY USA",
 keywords = "axiomref"
            — ignore —
\bibitem[Dalmas 92]{Dal92} Dalmas, S.
  title = "A polymorphic functional language applied to symbolic computation",
In Wang [Wan92] pp369-375 ISBN 0-89791-489-9 (soft cover) 0-89791-490-2
(hard cover) LCCN QA76.95.I59 1992
 keywords = "axiomref",
            — axiom.bib —
@misc{Daly88,
  author = "Daly, Timothy",
 title = "Axiom in an Educational Setting, Axiom course slide deck",
 year = "1988",
 month = "January",
 keywords = "axiomref"
            — ignore —
TPDHERE
\bibitem[Daly 02]{Dal02} Daly, Timothy
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title = "Axiom as open source",
SIGSAM Bulletin (ACM Special Interest Group
on Symbolic and Algebraic Manipulation) 36(1) pp28-?? March 2002
CODEN SIGSBZ ISSN 0163-5824
keywords = "axiomref",

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\bibitem[Daly 03]{Dal03} Daly, Timothy
 title = "The Axiom Wiki Website",
 url = "http://axiom.axiom-developer.org",
 keywords = "axiomref",

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\bibitem[Daly 06]{Dal06} Daly, Timothy
 title = "Axiom Volume 1: Tutorial",
Lulu, Inc. 860 Aviation Parkway,
Suite 300, Morrisville, NC 27560 USA, 2006 ISBN 141166597X 287pp
 url = "http://www.lulu.com/content/190827",
 keywords = "axiomref",

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\bibitem[Daly 09]{Dal09} Daly, Timothy
 title = "The Axiom Literate Documentation",
 url = "http://axiom-developer.org/axiom-website/documentation.html",
 keywords = "axiomref",

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\bibitem[Daly 13]{Dal13} Daly, Timothy
title = "Literate Programming in the Large'",

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  url = "http://conf.writethedocs.org",
 url2 = "http://daly.axiom-developer.org",
  video = "http://www.youtube.com/watch?v=AvOPQDVTP4A",
  keywords = "axiomref",
           - ignore -
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 title = "What can SCRATCHPAD/370 do?",
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 keywords = "axiomref",
            — ignore —
\bibitem[Davenport 80]{Dav80} Davenport, J.H.; Jenks, R.D.
  title = "MODLISP -- an Introduction",
Proc LISP80, 1980, and IBM RC8357 Oct 1980
 keywords = "axiomref",
           — ignore —
\bibitem[Davenport 84]{DGJ84} Davenport, J.; Gianni, P.; Jenks, R.;
Miller, V.; Morrison, S.; Rothstein, M.; Sundaresan, C.; Sutor, R.;
Trager, B.
  title = "Scratchpad",
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 keywords = "axiomref",
           — ignore —
\bibitem[Davenport 84a]{Dav84a} Davenport, James H.
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title = "A New Algebra System",

\bibitem[Davenport 88]{DST88} Davenport, J.H.; Siret, Y.; Tournier, E.
Computer Algebra: Systems and Algorithms for Algebraic Computation.
Academic Press, New York, NY, USA, 1988, ISBN 0-12-204232-9
 url = "http://staff.bath.ac.uk/masjhd/masternew.pdf",
 paper = "DST88.pdf",
 keywords = "axiomref",

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\bibitem[Davenport 14]{Dav14} Davenport, James H.
 title = "Computer Algebra textbook",
 url = "http://staff.bath.ac.uk/masjhd/JHD-CA.pdf",
 paper = "Dav14.pdf",
 keywords = "axiomref",

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\bibitem[Davenport 89]{Dav89} Davenport, J.H. (ed)
EUROCAL '87 European Conference on Computer Algebra Proceedings
Springer-Verlag, Berlin, Germany / Heidelberg, Germany / London,

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keywords = "axiomref",
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  title = "Scratchpad's view of algebra I: Basic commutative algebra",
In Miola [Mio90], pp40-54. ISBN 0-387-52531-9 (New York),
3-540-52531-9 (Berlin). LCCN QA76.9.S88I576 1990 also in AXIOM Technical
Report, ATR/1, NAG Ltd., Oxford, 1992
  keywords = "axiomref",
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— axiom.bib —

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@inproceedings{Dave91,
  author = "Davenport, J. H. and Gianni, P. and Trager, B. M.",
  title = "Scratchpad's View of Algebra II:
           A Categorical View of Factorization",
  booktitle = "Proc. 1991 Int. Symp. on Symbolic and Algebraic Computation",
  series = "ISSAC '91",
  year = "1991",
  isbn = "0-89791-437-6",
  location = "Bonn, West Germany",
  pages = "32--38",
  numpages = "7",
  url = "http://doi.acm.org/10.1145/120694.120699",
  doi = "10.1145/120694.120699",
  acmid = "120699",
  publisher = "ACM",
  address = "New York, NY, USA",
  keywords = "axiomref",
  paper = "Dave91.pdf",
  abstract = "
    This paper explains how Scratchpad solves the problem of presenting a
```

This paper explains how Scratchpad solves the problem of presenting categorical view of factorization in unique factorization domains, i.e. a view which can be propagated by functors such as SparseUnivariatePolynomial or Fraction. This is not easy, as the constructive version of the classical concept of UniqueFactorizationDomain cannot be so propagated. The solution adopted is based largely on Seidenberg's conditions (F) and (P), but there are several additional points that have to be borne in mind to produce reasonably efficient algorithms in the required generality.

The consequence of the algorithms and interfaces presented in this paper is that Scratchpad can factorize in any extension of the integers or finite fields by any combination of polynomial, fraction and algebraic extensions: a capability far more general than any other computer algebra system possesses. The solution is not perfect: for example we cannot use these general constructions to factorize polyinmoals in $\scriptstyle \$ overline{ $Z[\$ is not a unique factorization domain, even though $\$ is not a unique factorization domain, even though $\$ overline{ $Z[\$ is, since it is a field. Of course, we can factor polynomials in $\$ overline{ $Z[\$ is, $Z[\$ is, $Z[\$ is] in $Z[\$

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 title = "Scratchpad's view of algebra II: A categorical view of factorization",
Technical Report TR4/92 (ATR/2) (NP2491), Numerical Algorithms Group, Inc.,
Downer's Grove, IL, USA and Oxford, UK, December 1992
 url = "http://www.nag.co.uk/doc/TechRep/axiomtr.html",
 keywords = "axiomref",

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 title = "The AXIOM system",
AXIOM Technical Report TR5/92 (ATR/3)
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 url = "http://www.nag.co.uk/doc/TechRep/axiomtr.html",
 keywords = "axiomref",

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Numerical Algorithms Group, Inc., Downer's
Grove, IL, USA and Oxford, UK December 1992
  url = "http://www.nag.co.uk/doc/TechRep/axiomtr.html",
  paper = "Dav92b.pdf",
  keywords = "axiomref",
  abstract = "
    Axiom is a computer algebra system superficially like many others, but
    fundamentally different in its internal construction, and therefore in
    the possibilities it offers to its users and programmers. In these
    lecture notes, we will explain, by example, the methodology that the
    author uses for programming substantial bits of mathematics in Axiom."
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\bibitem[Davenport 92c]{DT92} Davenport, J. H.; Trager, B. M.
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Algorithms Group, Inc., Downer's Grove, IL, USA and Oxford, UK,
December 1992.
  url = "http://www.nag.co.uk/doc/TechRep/axiomtr.html",
  keywords = "axiomref",
            — ignore —
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  title = "Primality testing revisited",
Technical Report TR2/93 (ATR/6)(NP2556) Numerical Algorithms Group, Inc.,
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  url = "http://www.nag.co.uk/doc/TechRep/axiomtr.html",
  keywords = "axiomref",
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\bibitem[Davenport (a)]{DFxx} Davenport, James; Faure, Christ\'ele
  title = "The Unknown in Computer Algebra",
  url = "http://axiom-wiki.newsynthesis.org/public/refs/TheUnknownInComputerAlgebra.pdf",
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paper = "DFxx.pdf",
  keywords = "axiomref",
  abstract = "
    Computer algebra systems have to deal with the confusion between
    ''programming variables'' and ''mathematical symbols''. We claim that
   they should also deal with ''unknowns'', i.e. elements whose values
    are unknown, but whose type is known. For examples x^p \le x
    is a symbol, but x^p = x  if x \in GF(p). We show how we have
    extended Axiom to deal with this concept."
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 title = "A New Algebra System",
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 paper = "Dav00.pdf",
 keywords = "axiomref",
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  title = "Computer Algebra",
  url = "http://staff.bath.ac.uk/masjhd/JHD-CA.pdf",
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  url = "http://www.cs.berkeley.edu/~fateman/papers/ding.ps",
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   Many standard functions, such as the logarithms and square root
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   lead computer algebra systems into various conundrums. We discuss how
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Computer algebra systems are large collections of routines for solving mathematical problems algorithmically, efficiently and above all, symbolically. The more advanced and rigorous computer algebra systems (for example, Axiom) use the concept of strong types based on order-sorted algebra and category theory to ensure that operations are only applied to expressions when they ''make sense''.

In cases where Axiom uses notions which are not covered by current mathematics we shall present new mathematics which will allow us to prove that all such cases are reducible to cases covered by the current theory. On the other hand, we shall also point out all the cases where Axiom deviates undesirably from the mathematical ideal. Furthermore we shall propose solutions to these deviations.

Strongly typed systems (especially of mathematics) become unusable unless the system can change the type in a way a user expects. We wish any change expected by a user to be automated, ''natural'', and unique. ''Coercions'' are normally viewed as ''natural type changing maps''. This thesis shall rigorously define the word ''coercion'' in the context of computer algebra systems.

We shall list some assumptions so that we may prove new results so that all coercions are unique. This concept is called "coherence".

We shall give an algorithm for automatically creating all coercions in type system which adheres to a set of assumptions. We shall prove that this is an algorithm and that it always returns a coercion when one exists. Finally, we present a demonstration implementation of this automated coerion algorithm in Axiom."

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In this paper the analysis of the data structures used in a symbolic computation system, called Kenzo, is undertaken. We deal with the specification of the inheritance relationship since Kenzo is an object-oriented system, written in CLOS, the Common Lisp Object System. We focus on a particular case, namely the relationship between simplicial sets and chain complexes, showing how the order-sorted algebraic specifications formalisms can be adapted, through the "'inheritance as coercion" metaphor, in order to model this Kenzo fragment."

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title = "Computer Algebra meets Automated Theorem Proving: Integrating Maple and PVS", TPHOLS 2001, Edinburgh

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We describe an interface between version 6 of the Maple computer algebra system with the PVS automated theorem prover. The interface is designed to allow Maple users access to the robust and checkable proof

environment of PVS. We also extend this environment by the provision of a library of proof strategies for use in real analysis. We demonstrate examples using the interface and the real analysis library. These examples provide proofs which are both illustrative and applicable to genuine symbolic computation problems."

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 url = "http://wstein.org/papers/icms/icms_2010.pdf",
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 Sage is a free, open source, self-contained distribution of
 mathematical software, including a large library that provides a
 unified interface to the components of this distribution. This library
 also builds on the components of Sage to implement novel algorithms
 covering a broad range of mathematical functionality from algebraic

combinatorics to number theory and arithmetic geometry."

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title = "Advances and trends in the design and construction of algebraic manipulation systems"
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    Some of the earliest computer algebra systems (CAS) looked like
    overloaded languages of the same era. FORMAC, PL/I FORMAC, Formula
    Algol, and others each took advantage of a pre-existing language base
    and expanded the notion of a numeric value to include mathematical
    expressions. Much more recently, perhaps encouraged by the growth in
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This paper makes three points. 1. It is easy to do overloading in Common Lisp, and show how to do it in detail. 2. Overloading per se provides an easy solution to some simple programming problems. We show how it can be used for a ''demonstration'' CAS. Other simple and plausible overloadings interact nicely with this basic system. 3. Not all goes so smoothly: we can view overloading as a case study and perhaps an object lesson since it fails to solve a number of fairly-well articulated and difficult design issues in CAS for which other approaches are preferable."

popularity of C++, we have seen a renewal of the use of overloading to

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implement a CAS.

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ISSN 0249-6399 Institut National De Recherche en Informatique et en
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 paper = "FDN00b.pdf",
 keywords = "axiomref",
 abstract = "
 One of the main strengths of computer algebra is being able to solve a

family of problems with one computation. In order to express not only one problem but a family of problems, one introduces some symbols which are in fact the parameters common to all the problems of the family.

The user must be able to understand in which way these parameters affect the result when he looks at the answer. Otherwise it may lead to completely wrong calculations, which when used for numerical applications bring nonsensical answers. This is the case in most current Computer Algebra Systems we know because the form of the answer is never explicitly conditioned by the values of the parameters. The user is not even informed that the given answer may be wrong in some cases then computer algebra systems can not be entirely trustworthy. We have introduced multi-valued expressions called {\sl conditional} expressions, in which each potential value is associated with a condition on some parameters. This is used, in particular, to capture the situation in integration, where the form of the answer can depend on whether certain quantities are positive, negative or zero. We show that it is also necessary when solving modular linear equations or deducing congruence conditions from complex expressions."

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 title = "Buchberger's algorithm and staggered linear bases",
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paper = "GM88.pdf",
keywords = "axiomref",

abstract = "

Buchberger's algorithm calculates Groebner bases of polynomial ideals. Its efficiency depends strongly on practical criteria for detecting superfluous reductions. Buchberger recommends two criteria. The more important one is interpreted in this paper as a criterion for detecting redundant elements in a basis of a module of syzygies. We present a method for obtaining a reduced, nearly minimal basis of that module. The simple procedure for detecting (redundant syzygies and)superfluous reductions is incorporated now in our installation of Buchberger's algorithm in SCRATCHPAD II and REDUCE 3.3. The paper concludes with statistics stressing the good computational properties of these installations."

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 title = "Algorithms For Computer Algebra",
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We present here some examples of using the ''Dynamic Constructible Closure'' program, which performs automatic case distinction in computations involving parameters over a base field \$K\$. This program is an application of the ''Dynamic Evaluation'' principle, which generalizes traditional evaluation and was first used to deal with algebraic numbers."

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We present hidden verification as a means to make the power of computational logic available to users of computer algebra systems while shielding them from its complexity. We have implemented in PVS a library of facts about elementary and transcendental function, and automatic procedures to attempt proofs of continuity, convergence and differentiability for functions in this class. These are called directly from Maple by a simple pipe-lined interface. Hence we are able to support the analysis of differential equations in Maple by direct calls to PVS for: result refinement and verification, discharge of verification conditions, harnesses to ensure more reliable differential equation solvers, and verifiable look-up tables."

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We report on some experiences with the general purpose Computer Algebra Systems (CAS) Axiom, Macsyma, Maple, Mathematica, MuPAD, and Reduce solving systems of polynomial equations and the way they present their solutions. This snapshot (taken in the spring of 1996) of the current power of the different systems in a special area concentrates on both CPU-times and the quality of the output."

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The SCRATCHPAD/1 system is designed to provide an interactive symbolic computational facility for the mathematician user. The system features a user language designed to capture the style and succinctness of mathematical notation, together with a facility for conveniently introducing new notations into the language. A comprehensive system library incorporates symbolic capabilities provided by such systems as SIN, MATHLAB, and REDUCE."

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 abstract = "

This thesis presents an algorithm for computing (one-sided) limits within a symbolic manipulation system. Computing limits is an important facility, as limits are used both by other functions such as the definite integrator and to get directly some qualitative information about a given function.

The algorithm we present is very compact, easy to understand and easy to implement. It overcomes the cancellation problem other algorithms suffer from. These goals were achieved using a uniform method, namely by expanding the whole function into a series in terms of its most rapidly varying subexpression instead of a recursive bottom up expansion of the function. In the latter approach exact error terms have to be kept with each approximation in order to resolve the cancellation problem, and this may lead to an intermediate expression

swell. Our algorithm avoids this problem and is thus suited to be implemented in a symbolic manipulation system."

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Scratchpad II is an abstract datatype language and system that is under development in the Computer Algebra Group, Mathematical Sciences Department, at the IBM Thomas J. Watson Research Center. Some features of APL that made computation particularly elegant have been borrowed. Many different kinds of computational objects and data structures are provided. Facilities for computation include symbolic integration, differentiation, factorization, solution of equations and linear algebra. Code economy and modularity is achieved by having polymorphic packages of functions that may create datatypes. The use of categories makes these facilities as general as possible."

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A new method is proposed for finding the logarithmic part of an integral over an algebraic function. The method uses $Gr{\normalcolor}$ bases and is easy to implement. It does not have the feature of finding a closed form of an integral whenever there is one. But it very often does, as we will show by a comparison with the built-in integrators of some computer algebra systems."

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One way in which mathematicians deal with infinite amounts of data is symbolic representation. A simple example is the quadratic equation $\label{eq:continuous} $$ (x = \frac{-b\pm 1}{2a}) $$$

a formula which uses symbolic representation to describe the solutions to an infinite class of equations. Most computer algebra systems can deal with polynomials with symbolic coefficients, but what if symbolic exponents are called for (e.g. $1+t^i$)? What if symbolic limits on summations are also called for, for example $[1+t+\dot s+t^i=\sum_{j=1}^{\infty}]$

The ''Scratchpad Concept'' is a theoretical ideal which allows the implementation of objects at this level of abstraction and beyond in a mathematically consistent way. The Axiom computer algebra system is an implementation of a major part of the Scratchpad Concept. Axiom (formerly called Scratchpad) is a language with extensible parameterized types and generic operators which is based on the notions of domains and categories. By examining some aspects of the Axiom system, the Scratchpad Concept will be illustrated. It will be shown how some complex problems in homologicial algebra were solved through the use of this system."

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This paper investigates some soundness conditions which have to be fulfilled in systems with coercions and generic operators. A result of Reynolds on unrestricted generic operators is extended to generic operators which obey certain constraints. We get natural conditions for such operators, which are expressed within the theoretic framework of category theory. However, in the context of computer algebra, there arise examples of coercions and generic operators which do not fulfil these conditions. We describe a framework -- relaxing the above conditions -- that allows distinguishing between cases of ambiguities which can be resolved in a quite natural sense and those which cannot. An algorithm is presented that detects such unresolvable ambiguities in expressions."

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 abstract = "

The Macsyma system arose out of research on mathematical software in the AI group at MIT in the 1960's. Algorithm development in symbolic integration and simplification arose out of the interest of people, such as the author, who were also mathematics students. The later development of algorithms for the GCD of sparse polynomials, for example, arose out of the needs of our user community. During various times in the 1970's the computer on which Macsyma ran was one of the most popular notes on the ARPANET. We discuss the attempts in the late 70's and the 80's to develop Macsyma systems that ran on popular computer architectures. Finally, we discuss the impact of the fundamental ideas in Macsyma on current research on large scale engineering systems."

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OpenMath is a widely recognized approach to the semantic markup of mathematics that is often used for communication between OpenMath compliant systems. The Aldor language has a sophisticated category-based type system that was specifically developed for the purpose of modelling mathematical structures, while the system itself supports the creation of small-footprint applications suitable for deployment as web services. In this paper we present our first results of how one may perform translations from generic OpenMath objects into values in specific Aldor domains, describing how the Aldor interfae domain ExpresstionTree is used to achieve this. We outline our Aldor implementation of an OpenMath translator, and describe an efficient extention of this to the Parser category. In addition, the Aldor service creation and invocation mechanism are explained. Thus we are in a position to develop and deploy mathematical web services whose descriptions may be directly derived from Aldor's rich type language."

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An Axiom front end is described, which is used to generate mathematical objects needed by one of the latest NAG routines, to be included in the Mark 17 version of the NAG Numerical library. This routine uses powerful techniques to find the solution to Hyperbolic Partial Differential Equations in conservation form and in one spatial dimension. These mathematical objects are non-trivial, requiring much mathematical knowledge on the part of the user, which is otherwise irrelvant to the physical problem which is to be solved. We discuss the individual mathematical objects, considering the mathematical theory which is relevant, and some of the problems which have been encountered and solved during the FORTRAN generation necessary to realise the object. Finally we display some of our results."

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The normal claim for mathematics is that all calculations are 100\% accurate and therefore one calculation can rely completely on the results of sub-calculations, hoever there exist {\sl Monte-Carlo} algorithms which are often much faster than the equivalent deterministic ones where the results will have a prescribed probability (presumably small) of being incorrect. However there has been little discussion of how such algorithms can be used as building blocks in Computer Algebra. In this paper we describe how the computational category theory which is the basis of the type structure used in the Axiom computer algebra system may be extended to cover probabilistic algorithms, which use Monte-Carlo techniques. We follow this with a specific example which uses Straight Line Program representation."

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    This paper presents a new encoding scheme for real algebraic number
   manipulations which enhances current Axiom's real closure. Algebraic
   manipulations are performed using different instantiations of
    sub-resultant-like algorithms instead of Euclidean-like algorithms.
    We use these algorithms to compute polynomial gcds and Bezout
    relations, to compute the roots and the signs of algebraic
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  title = "Does Axiom Solve Systems of O.D.E's Like Mathematica?",
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    If I were demonstrating Axiom and were asked this question, my reply
    would be ''No, but I am not sure that this is a bad thing''. And I
    would illustrate this with the following example.
    Consider the following system of O.D.E.'s
    \begin{array}{rcl}
    \frac{dx_1}{dt} & = & \left(1+\frac{cos t}{2+sin t}\right)x_1
    \frac{dx_2}{dt} & = & x_1 - x_2
    \end{array}
    This is a very simple system: $x_1$ is actually uncoupled from $x_2$"
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  paper = "Rio92.pdf",
  keywords = "axiomref",
  abstract = "
    Real algebraic numbers appear in many Computer Algebra problems. For
    instance the determination of a cylindrical algebraic decomposition
    for an euclidean space requires computing with real algebraic numbers.
    This paper describes an implementation for computations with the real
    roots of a polynomial. This process is designed to be recursively
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author = "Saunders, B. David",
title = "A Survey of Available Systems",
journal = "SIGSAM Bull.",
issue_date = "November 1980",
volume = "14",
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month = "November",
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doi = "10.1145/1089235.1089237",
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address = "New York, NY, USA",
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abstract = "

An introduction to the formal theory of partial differential equations is given emphasizing the properties of involutive symbols and equations. An algorithm to complete any differential equation to an involutive one is presented. For an involutive equation possible values for the number of arbitrary functions in its general solution are determined. The existence and uniqueness of solutions for analytic equations is proven. Applications of these results include an analysis of symmetry and reduction methods and a study of gauge systems. It is show that the Dirac algorithm for systems with constraints is closely related to the completion of the equation of motion to an involutive equation. Specific examples treated comprise the Yang-Mills Equations, Einstein Equations, complete and Jacobian systems, and some special models in two and three dimensions. To facilitate the involved tedious computations an environment for geometric approaches to differential equations has been developed in the computer algebra system Axiom. The appendices contain among others brief introductions into Carten-K{\"a}hler Theory and Janet-Riquier Theory."

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We present an Axiom environment called JET for geometric computations with partial differential equations within the framework of the jet bundle formalism. This comprises expecially the completion of a given differential equation to an involutive one according to the Cartan-Kuranishi Theorem and the setting up of the determining system for the generators of classical and non-classical Lie symmetries. Details of the implementations are described and applications are given. An appendix contains tables of all exported functions."

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 $\label{thm:condition} $$ \left[Seiler, W.M.; Calmet, J. \right] $$$

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paper = "SC95.pdf",
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JET is an environment within the computer algebra system Axiom to perform such computations. The current implementation emphasises the two key concepts involution and symmetry. It provides some packages for the completion of a given system of differential equations to an equivalent involutive one based on the Cartan-Kuranishi theorem and for setting up the determining equations for classical and non-classical point symmetries."

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We present a theoretical foundation for studying parametric systesm of linear equations and prove an efficient algorithm for identifying all parametric values (including degnerate cases) for which the system is consistent. The algorithm gives a small set of regimes where for each regime, the solutions of the specialized systems may be given uniformly. For homogeneous linear systems, or for systems were the right hand side is arbitrary, this small set is irredunant. We discuss in detail practical issues concerning implementations, with particular emphasis on simplification of results. Examples are given based on a close implementation of the algorithm in SCRATCHPAD II. We also give a complexity analysis of the Gaussian elimination method and compare that with our algorithm."

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   The Scratchpad II system is an abstract datatype programming language,
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a compiler for the language, a library of packages of polymorphic functions and parameterized abstract datatypes, and an interpreter that provides sophisticated type inference and coercion facilities. Although originally designed for the implementation of symbolic mathematical algorithms, Scratchpad II is a general purpose programming language. This paper discusses aspects of the implementation of the interpreter and how it attempts to provide a user friendly and relatively weakly typed front end for the strongly typed programming language."

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title = "Logic and dependent types in the Aldor Computer Algebra System",
paper = "Tho00.pdf",
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We show how the Aldor type system can represent propositions of first-order logic, by means of the 'propositions as types' correspondence. The representation relies on type casts (using pretend) but can be viewed as a prototype implementation of a modified type system with {\sl type evaluation} reported elsewhere. The logic is used to provide an axiomatisation of a number of familiar Aldor categories as well as a type of vectors."

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  paper = "TTxx.pdf",
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  abstract = "
    This paper introduces the \verb|Aldor--| language, which is a
    functional programming language with dependent types and a powerful,
    type-based, overloading mechanism. The language is built on a subset
    of Aldor, the 'library compiler' language for the Axiom computer
    algebra system. \verb|Aldor--| is designed with the intention of
    incorporating logical reasoning into computer algebra computations.
    The paper contains a formal account of the semantics and type system
    of \verb|Aldor--|; a general discussion of overloading and how the
    overloading in \verb|Aldor--| fits into the general scheme; examples
    of logic within \verb | Aldor-- | and notes on the implementation of the
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Algorithms for computing integral bases of an algebraic function field are implemented in some computer algebra systems. They are used e.g. for the integration of algebraic functions. The method used by Maple 5.2 and AXIOM is given by Trager in [Trag84]. He adapted an algorithm of Ford and Zassenhaus [Ford, 1978], that computes the ring of integers in an algebraic number field, to the case of a function field.

It turns out that using algebraic geometry one can write a faster algorithm. The method we will give is based on Puiseux expansions. One cas see this as a variant on the Coates' algorithm as it is described in [Davenport, 1981]. Some difficulties in computing with Puiseux expansions can be avoided using a sharp bound for the number of terms required which will be given in Section 3. In Section 5 we derive which denominator is needed in the integral basis. Using this result 'intermediate expression swell' can be avoided.

The Puiseux expansions generally introduce algebraic extensions. These extensions will not appear in the resulting integral basis." $\frac{1}{2} \left(\frac{1}{2} \right) \left(\frac{1}{2} \right)$

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Computer algebra systesm often produce large expressions involving complicated algebraic numbers. In this paper we study variations of the {\tt polred} algorithm that can often be used to find better representations for algebraic numbers. The main new algorithm

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presented here is an algorithm that treats the same problem for the
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  keywords = "axiomref",
  abstract = "
    An important feature of modern computer algebra systems is the support
    of a rich type system with the possibility of type inference. Basic
    features of such a type system are polymorphism and coercion between
    types. Recently the use of order-sorted rewrite systems was proposed
    as a general framework. We will give a quite simple example of a
    family of types arising in computer algebra whose coercion relations
    cannot be captured by a finite set of first-order rewrite rules."
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\bibitem[Weber 92b]{Webe92b} Weber, Andreas
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  paper = "Webe92b.pdf",
  keywords = "axiomref",
  abstract = "
   Most existing computer algebra systems are pure symbol manipulating
```

systems without language support for the occuring types. This is mainly due to the fact taht the occurring types are much more complicated than in traditional programming languages. In the last decade the study of type systems has become an active area of research. We will give a proposal for a type system showing that several problems for a type system of a symbolic computation system can be solved by using results of this research. We will also provide a variety of examples which will show some of the problems that remain and that will require further research."

— ignore —

\bibitem[Weber 93b]{Webe93b} Weber, Andreas
 title = "Type Systems for Computer Algebra",
 url = "http://cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber93b.pdf",
 paper = "Webe93b.pdf",
 keywords = "axiomref",
 abstract = "

We study type systems for computer algebra systems, which frequently correspond to the ''pragmatically developed'' typing constructs used in AXIOM. A central concept is that of {\sl type classes} which correspond to AXIOM categories. We will show that types can be syntactically described as terms of a regular order-sorted signature if no type parameters are allowed. Using results obtained for the functional programming language Haskell we will show that the problem of {\sl type inference} is decidable. This result still holds if higher-order functions are present and {\sl parametric polymorphism} is used. These additional typing constructs are useful for further extensions of existing computer algebra systems: These typing concepts can be used to implement category theoretic constructs and there are many well known constructive interactions between category theory and algebra."

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\bibitem[Weber 94]{Web94} Weber, Andreas
 title = "Algorithms for Type Inference with Coercions",
ISSAC 94 ACM 0-89791-638-7/94/0007
 paper = "Web94.pdf",
 keywords = "axiomref",
 abstract = "
 This paper presents algorithms that perform a type inference for a

type system occurring in the context of computer algebra. The type system permits various classes of coercions between types and the algorithms are complete for the precisely defined system, which can be seen as a formal description of an important subset of the type system supported by the computer algebra program Axiom.

Previously only algorithms for much more restricted cases of coercions have been described or the frameworks used have been so general that the corresponding type inference problems were known to be undecidable."

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\bibitem[Weber 95]{Webe95} Weber, A.
 title = "On coherence in computer algebra",
 url = "http://cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber94e.pdf",
 paper = "Webe95.pdf",
 keywords = "axiomref",
 abstract = "
 Modern computer algebra systems (e.g. AXIOM) support a rich type

Modern computer algebra systems (e.g. AXIOM) support a rich type system including parameterized data types and the possibility of implicit coercions between types. In such a type system it will be frequently the case that there are different ways of building coercions between types. An important requirement is that all coercions between two types coincide, a property which is called {\sl coherence}. We will prove a coherence theorem for a formal type system having several possibilities of coercions covering many important examples. Moreover, we will give some informal reasoning why the formally defined restrictions can be satisfied by an actual system."

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factor of \$p-1\$."

\bibitem[Weber 96]{Webe96} Weber, Andreas
 title = "Computing Radical Expressions for Roots of Unity",
 url = "http://cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/Weber96a.pdf",
 paper = "Webe96.pdf",
 keywords = "axiomref",
 abstract = "
 We present an improvement of an algorithm given by Gauss to compute a
 radical expression for a \$p\$-th root of unity. The time complexity of
 the algorithm is \$O(p^3m^6log p)\$, where \$m\$ is the largest prime

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\bibitem[Weber 99]{Webe99} Weber, Andreas
 title = "Solving Cyclotomic Polynomials by Radical Expressions",
 url = "http://cg.cs.uni-bonn.de/personal-pages/weber/publications/pdf/WeberA/WeberKeckeisen99a
 paper = "Webe99.pdf",
 keywords = "axiomref",
 abstract = "
 We describe a Maple package that allows the solution of cyclotomic
 polynomials by radical expressions. We provide a function that is an
 extension of the Maple {\sl solve} command. The major algorithmic
 ingredient of the package is an improvement of a method due to Gauss
 which gives radical expressions for roots of unity. We will give a

summary for computations up to degree 100, which could be done within

a few hours of cpu time on a standard workstation."

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\bibitem[Wei-Jiang 12]{WJ12} Wei-Jiang
 title = "Top free algebra System",
 url = "http://wei-jiang.com/it/software/top-free-algebra-system-bye-mathematica-bye-maple",
 keywords = "axiomref",

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@misc{West99a,
 author = "Wester, Michael J.",
 title = "A Critique of the Mathematical Abilities of CA Systems",
 year = "1999",
 url = "http://math.unm.edu/~wester/cas/book/Wester.pdf",
 url2 = "http://math.unm.edu/~wester/cas_review.html",
 paper = "West99a.pdf",
 abstract =
 "Computer algebra systems (CASs) have become an essential computational
 tool in the last decade. General purpose CASs, which are designed to

solve a wide variety of problems, have gained special prominence. In

this chapter, the capabilities of seven major general purpose CASs (Axiom, Derive, Macsyma, Maple, Mathmatica, MuPAD and Reduce) are reviewed on 542 short problems covering a broad range of (primarily) symbolic mathematics."

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 title = "Computer Algebra Systems",
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\bibitem[Wexelblat 87]{Wex87} Wexelblat, Richard L. (ed)
Proceedings of the SIGPLAN '87 Symposium on
Interpreter and Interpretive Techniques, St. Paul, Minnesota, June 24-26, 1987
ACM Press, New York, NY 10036, USA, 1987 ISBN 0-89791-235-7
LCCN QA76.7.S54 v22:7 SIGPLAN Notices, vol 22, no 7 (July 1987)
 keywords = "axiomref",

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\bibitem[Wityak 87]{Wit87} Wityak, Sandra
 title = "Scratchpad II Newsletter",
Volume 2, Number 1, Nov 1987
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  url = "http://www.softwarepresentation.org/projects/LISP/common_lisp_family",
  keywords = "axiomref",
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\bibitem[Yap 00]{Yap00} Yap, Chee Keng
  title = "Fundamental Problems of Algorithmic Algebra",
Oxford University Press (2000) ISBNO-19-512516-9
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  title = "Literate Programming Tools Implemented in ANSI Common Lisp",
  url = "http://brlcad.org/~starseeker/cl-web-v0.8.lisp.pamphlet",
  keywords = "axiomref",
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\bibitem[Yun 83]{Yun83} Yun, David Y.Y.
  title = "Computer Algebra and Complex Analysis",
Computational Aspects of Complex Analysis pp379-393
D. Reidel Publishing Company H. Werner et. al. (eds.)
  keywords = "axiomref",
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@article{Abla98,
  author = "Ablamowicz, Rafal",
  title = "Spinor Representations of Clifford Algebras: A Symbolic Approach",
  journal = "Computer Physics Communications",
  volume = "115",
  number = "2-3",
  month = "December",
  year = "1998",
  pages = "510-535"
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}
            — axiom.bib —
@article{Abra06,
  author = "Abramov, Sergey A.",
  title = "{In Memory of Manuel Bronstein}",
  journal = "Programming and Computer Software",
  volume = "32",
  number = "1",
  pages = "56-58",
 publisher = "Pleiades Publishing Inc",
 year = "2006",
 paper = "Abra06.pdf",
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\bibitem[Abramowitz 64]{AS64} Abramowitz, Milton; Stegun, Irene A.
 title = "Handbook of Mathematical Functions",
(1964) Dover Publications, NY ISBN 0-486-61272-4
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  title = "Handbook of Mathematical Functions",
Dover Publications. (1968)
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@book{Altm05,
  author = "Altmann, Simon L.",
  title = "Rotations, Quaternions, and Double Groups",
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publisher = "Dover Publications, Inc.",
 year = "2005",
  isbn = "0-486-44518-6"
            — ignore —
\bibitem[Ames 77]{Ames77} Ames W F
  title = "Nonlinear Partial Differential Equations in Engineering",
Academic Press (2nd Edition). (1977)
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 title = "Algorithm 644: A Portable Package for Bessel Functions of a Complex Argument and Nonnegative Ord
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  title = "On Mechanical Quantifier Elimination for Elementary Algebra and Geometry",
J. Symbolic Computation 5, 237-259 (1988)
  url = "http://http://www.sciencedirect.com/science/article/pii/S0747717188800142/pdf?md5=62052
  paper = "Arno88.pdf",
  abstract = "
    We give solutions to two problems of elementary algebra and geometry:
    (1) find conditions on real numbers $p$, $q$, and $r$ so that the
    polynomial function f(x)=x^4+px^2+qx+r is nonnegative for all real
    $x$ and (2) find conditions on real numbers $a$, $b$, and $c$ so that
    the ellipse \frac{(x-e)^2}{q^2}+\frac{y^2}{b^2}-1=0 lies inside the
    unit circle y^2+x^2-1=0. Our solutions are obtained by following the
    basic outline of the method of quantifier elimination by cylindrical
    algebraic decomposition (Collins, 1975), but we have developed, and
    have been considerably aided by, modified versions of certain of its
    steps. We have found three equally simple but not obviously equivalent
    solutions for the first problem, illustrating the difficulty of
    obtaining unique ''simplest'' solutions to quantifier elimination
    problems of elementary algebra and geometry."
            — axiom.bib —
@article{Aubr99,
  author = "Aubry, Phillippe and Lazard, Daniel and {Moreno Maza}, Marc",
  title = "On the Theories of Triangular Sets",
  year = "1999",
 pages = "105-124",
  journal = "Journal of Symbolic Computation",
  volume = "28",
  url = "http://www.csd.uwo.ca/~moreno/Publications/Aubry-Lazard-MorenoMaza-1999-JSC.pdf",
  papers = "Aubr99.pdf",
  abstract = "
   Different notions of triangular sets are presented. The relationship
    between these notions are studied. The main result is that four
    different existing notions of {\sl good} triangular sets are
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\bibitem[Aubry 96]{Aub96} Aubry, Philippe; Maza, Marc Moreno title = "Triangular Sets for Solving Polynomial Systems: a Comparison of Four Methods",

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  paper = "Aub96.ps",
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   Four methods for solving polynomial systems by means of triangular
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  title = "Sturm-Liouville Eigenvalues via a Phase Function",
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  title = "Pade Approximants",
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title = "Axiom Architecture",
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keywords = "axiomref",
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\bibitem[Banks 68]{BK68} Banks D O; Kurowski I
  title = "Computation of Eigenvalues of Singular Sturm-Liouville Systems",
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\bibitem[Bard 74]{Bard74} Bard Y
  title = "Nonlinear Parameter Estimation",
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  title = "An Improved Algorithm for Discrete $11_1$ Linear Approximation",
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\bibitem[Barrodale 74]{BR74} Barrodale I; Roberts F D K
 title = "Solution of an Overdetermined System of Equations in the $11_1-norm$.",
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 title = "Products of polynomials and a priori estimates for coefficients in polynomial decompositions: a
J. Symbolic Computation (1992) 13, 463-472
 paper = "Bea92.pdf",
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\bibitem[Beauzamy 93]{Bea93} Beauzamy, Bernard; Trevisan, Vilmar;
Wang, Paul S.
 title = "Polynomial Factorization: Sharp Bounds, Efficient Algorithms",
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@article{Bert95,
  author = "Bertrand, Laurent",
  title = "Computing a hyperelliptic integral using arithmetic in the
          jacobian of the curve",
  journal = "Applicable Algebra in Engineering, Communication and Computing",
  volume = "6",
  pages = "275-298",
 year = "1995",
  abstract = "
   In this paper, we describe an efficient algorithm for computing an
   elementary antiderivative of an algebraic function defined on a
   hyperelliptic curve. Our algorithm combines B.M. Trager's integration
   algorithm and a technique for computing in the Jacobian of a
   hyperelliptic curve introduced by D.G. Cantor. Our method has been
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}
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title = "Design of the Stiff Integrators in the NAG Library",

Technical Report. TR14/87 NAG. (1987)

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 title = "Note on Irreducibility Testing",
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 title = "Symbolic Rewriting Techniques",
Progress in Computer Science and Applied Logic 15, Birkhauser-Verlag, Basel
ISBN 3-7643-5901-3 (1998)

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\bibitem[Bronstein 88]{Bro88} Bronstein, Manual
 title = "The Transcendental Risch Differential Equation",
J. Symbolic Computation (1990) 9, pp49-60 Feb 1988
IBM Research Report RC13460 IBM Corp. Yorktown Heights, NY
 url = "http://www.sciencedirect.com/science/article/pii/S0747717108800065",
 paper = "Bro88.pdf",
 abstract = "

We present a new rational algorithm for solving Risch differential equations in towers of transcendental elementary extensions. In contrast to a recent algorithm by Davenport we do not require a progressive reduction of the denominators involved, but use weak normality to obtain a formula for the denominator of a possible solution. Implementation timings show this approach to be faster than a Hermite-like reduction."

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@techreport{Bron98,
  author = "Bronstein, Manuel",
  title = "The lazy hermite reduction",
  type = "Rapport de Recherche",
 number = "RR-3562",
  year = "1998",
  institution = "French Institute for Research in Computer Science",
  paper = "Bron98.pdf",
  abstract = "
    The Hermite reduction is a symbolic integration technique that reduces
    algebraic functions to integrands having only simple affine
    poles. While it is very effective in the case of simple radical
    extensions, its use in more general algebraic extensions requires the
    precomputation of an integral basis, which makes the reduction
    impractical for either multiple algebraic extensions or complicated
    ground fields. In this paper, we show that the Hermite reduction can
    be performed without {\sl a priori} computation of either a primitive
    element or integral basis, computing the smallest order necessary for
    a particular integrand along the way."
}
            — axiom.bib —
@misc{Bro98b,
  author = "Bronstein, Manuel",
  title = "Symbolic Integration Tutorial",
  series = "ISSAC'98",
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  address = "INRIA Sophia Antipolis",
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Ph.D Thesis, Univ. Delaware (1999)
  url = "http://www.usna.edu/Users/cs/wcbrown/research/thesis.ps.gz",
 paper = "Brow99.pdf",
  abstract = "
```

The CAD-based quantifier elimination algorithm takes a formula from the elementary theory of real closed fields as input, and constructs a CAD of the space of the formula's unquantified variables. This decomposition is truth invariant with respect to the input formula, meaning that the formula is either identically true or identically false in each cell of the decomposition. The method determines the truth of the input formula for each cell of the CAD, and then uses the CAD to construct a solution formula — a quantifier free formula that is equivalent to the input formula. This final phase of the algorithm, the solution formula construction phase, is the focus of this thesis.

An optimal solution formula construction algorithm would be {\sl complete} -- i.e. applicable to any truth-invariant CAD, would be {\sl efficient}, and would produce {\sl simple} solution formulas. Prior to this thesis, no method was available with even two of these three properties. Several algorithms are presented, all addressing problems related to solution formula construction. In combination, these provide an efficient and complete method for constructing solution formulas that are simple in a variety of ways.

Algorithms presented in this thesis have been implemented using the SACLIB library, and integrated into QEPCAD, a SACLIB-based implementation of quantifier elimination by CAD. Example computations based on these implementations are discussed."

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\bibitem[Brown 02]{Brow02} Brown, Christopher W.

title = "QEPCAD B -- A program for computing with semi-algebraic sets using CADs", paper = "Brow02.pdf", abstract = " $^{\circ}$

This report introduces QEPCAD B, a program for computing with real algebraic sets using cylindrical algebraic decomposition (CAD). QEPCAD B both extends and improves upon the QEPCAD system for quantifier elimination by partial cylindrical algebraic decomposition written by Hoon Hong in the early 1990s. This paper briefly discusses some of the improvements in the implementation of CAD and quantifier elimination vis CAD, and provides somewhat more detail on extensions to the system that go beyond quantifier elimination. The author is responsible for most of the extended features of QEPCAD B, but improvements to the basic CAD implementation and to the SACLIB library on which QEPCAD is based are the results of many people's work."

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@article{Burg74,
  author = "William H. Burge",
 title = "Stream Processing Functions",
  year = "1974",
 month = "January",
  journal = "IBM Journal of Research and Development",
  volume = "19",
  issue = "1",
  pages = "12-25",
  papers = "Burg74.pdf",
  abstract = "
    One principle of structured programming is that a program should be
    separated into meaningful independent subprograms, which are then
    combined so that the relation of the parts to the whole can be clearly
    established. This paper describes several alternative ways to compose
    programs. The main method used is to permit the programmer to denote
    by an expression the sequence of values taken on by a variable. The
    sequence is represented by a function called a stream, which is a
    functional analog of a coroutine. The conventional while and for loops
    of structured programming may be composed by a technique of stream
    processing (analogous to list processing), which results in more
```

structured programs than the orignals. This technique makes it possible to structure a program in a natural way into its logically

separate parts, which can then be considered independently."

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  title = "Curve Fitting with a Digital Computer",
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  title = "Atlas of Finite Groups",
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The State of the Art in Numerical Analysis. (ed D A H Jacobs)
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We define a categorical framework, based on the notion of {\sl sketch}, for specification and evaluation in the sense of algebraic specifications and algebraic programming. This framework goes far beyond our initial motivations, which was to specify computation with algebraic numbers. We begin by redefining sketches in order to deal explicitly with programs. Expressions and terms are carefully defined and studied, then {\sl quasi-projective sketches} are introduced. We describe {\sl static evaluation} in these sketches: we propose a rigorous basis for evaluation in the corresponding structures. These structures admit an initial model, but are not necessarily equational. In Part II (Duval and Reynaud 1994), we study a more general process, called {\sl dynamic evaluation}, for structures that may have no initial model."

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necessary, algebraic numbers. But in many circumstances an approximate
result is more likely to be of use. Furthermore, it is plausible that
it would be more useful to solve the problem to allow definite
integration, or introduce additional parameters so that we can solve
multiple definite integrations. How can a computer algebra system
best answer the more useful questions? Finally, what if the integrand
is not a ratio of polynomials, but something more challenging?"
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   relationship between mathematical domains of computation. To use these
   systems interactively, however, the user should be freed of type
   information. A type inference mechanism determines the appropriate
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function to call. All known models which allow to define a semantics for type inference cannot express the rich 'mathematical' type

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\ensuremath{\mbox{[e^x[exp(1/x+e^{-x})-exp(1/x)],\quad \hline}} \
```

In this example, if the sum is expanded in powers of 1/x, the expansion always yields $0(x^{-k})$, and this is not enough to conclude.

In 1990, J.Shackell found an algorithm that solved both these problems for the case of \$exp-log\$ functions, i.e. functions build by recursive application of exponential, logarithm, algebraic extension and field operations to one variable and the rational numbers. D. Gruntz and G. Gonnet propose a slightly different algorithm for exp-log functions. Extensions to larger classes of functions are also discussed."

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This paper proposes a fast algorithm for computing multiplicative inverses in \$GF(2^m)\$ using normal bases. Normal bases have the following useful property: In the case that an element \$x\$ in \$GF(2^m)\$ is represented by normal bases, \$2^k\$ power operation of an element x in $GF(2^m)$ can be carried out by k times cyclic shift of its vector representation. C.C. Wang et al. proposed an algorithm for computing multiplicative inverses using normal bases, which requires (m-2) multiplications in $GF(2^m)$ and (m-1) cyclic shifts. The fast algorithm proposed in this paper also uses normal bases, and computes multiplicative inverses iterating multiplications in $GF(2^m)$. It requires at most $2[\log_2(m-1)]$ multiplications in $GF(2^m)$ and (m-1) cyclic shifts, which are much less than those required in Wang's method. The same idea of the proposed fast algorithm is applicable to the general power operation in \$GF(2^m)\$ and the computation of multiplicative inverses in $GF(q^m)$ \$(q=2^n)\$."

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We describe the development of a repository of mathematical knowledge based on transformation rules. The specific mathematical problem is indefinite integration. It is important that the repository be not

confused with a look-up table. The database of transformation rules is at present encoded in Mathematica, but this is only one convenient form of the repository, and it could be readily translated into other formats. The principles upon which the set of rules is compiled is described. One important principle is minimality. The benefits of the approach are illustrated with examples, and with the results of comparisons with other approaches."

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 let \$L\$ be a linear differential operator with polynomial
 coefficients. We show that there is an isomorphism of differential
 operators \${\bf D_\alpha}\$ and an integral transform \${\bf H_\alpha}\$
 (called the Hermite transform) on functions for which \$({\bf
 D_\alpha}{\bf L})f(x)=0\$ implies \${\bf L}{\bf H_alpha}(f)(x)=0\$. We
 present an algorithm that computes the Hermite transform of a rational
 function and use it to find \$n+1\$ linearly independent solutions of
 \${\bf L}y=0\$ when \$({\bf D_\alpha}{\bf L})f(x)=0\$ has a rational
 solution with \$n\$ distinct finite poles."

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    This paper presents an optimized method for factoring multivariate
    polynomials over algebraic extension fields which defined by an
    irreducible ascending set. The basic idea is to convert multivariate
    polynomials to univariate polynomials and algebraic extensions fields
    to algebraic number fields by suitable integer substitutions, then
    factorize the univariate polynomials over the algebraic number fields.
    Finally, construct multivariate factors of the original polynomial by
    Hensel lemma and TRUEFACTOR test. Some examples with timing are
    included."
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   MACSYMA is a system for symbolic manipulation of algebraic expressions
    which is being developed at Project MAC, M.I.T. This paper discusses
    its philosophy, goals, and current achievements.
    Drawing on the past work of Maring, Moses, and Engelman, it extends
    the capabilities of automated algebraic manipulation systems in
    several areas, including
    a) limit calculations
   b) symbolic integration
    c) solution of equations
    d) canonical simplification
    e) user-level pattern matching
    f) user-specified expression manipulation
```

g) programming and bookkeeping assistance

MACSYMA makes extensive use of the power of its rational function subsystem. The facilities derived from this are discussed in considerable detail.

An appendix briefly notes some highlights of the overall system."

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    The $q$-Engle Expansion is an algorithm that leads to unique series
    expansions of $q$-series. Various examples related to classical
    partition theorems, including the Rogers-Ramanujan identities together
    with the elegant generalization found by Garrett, Ismail and Stanton,
    have been described recently. The object of this paper is to present
    the computer algebra package Engel, written in Mathematics, that has
    already played a signiicant role in this work. The package now is made
    freely available via the web and should help to intensify research in
    this new branch of q\-series theory. Among various illustrative
    examples we present a new infinite Rogers-Ramanujan type family that
    has been discovered by using the package."
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   We give algorithms for the computation of the $d$-th digit of certain
   transcendental numbers in various bases. These algorithms can be
    easily implemented (multiple precision arithmetic is not needed),
   require virtually no memeory, and feature run times that scale nearly
    linearly with the order of the digit desired. They make it feasible to
    compute, for example, the billionth binary digit of log(2) or $\pi$ on
    a modest work station in a few hours run time.
   We demonstrate this technique by computing the ten billionth
    hexadecimal digit of $\pi$, the billionth hexadecimal digits of
    $\pi^2$, log(2), and log${}^2$(2), and the ten billionth decimal digit
    of log(9/10).
   These calculations rest on the observation that very special types of
    identities exist for certain numbers like $\pi$, $\pi^2$, log(2) and
    log${}^2$. These are essentially polylogarithmic ladders in an integer
    base. A number of these identities that we derive in this work appear
    to be new, for example the critical identity for $\pi$:
    \left[ \phi_{i=\sum_{i=0}^{i}} \right] 
    \frac{4}{8i+1}-\frac{2}{8i+4}-\frac{1}{8i+5}-\frac{1}{8i+6}\right)}\]"
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 abstract =
  \verb"OpenDreamKit will deliver a flexible toolkit enabling research groups
   to set up Virtual Research Environments, customised to meet the varied
   needs of research projects in pure mathematics and applications, and
```

supporting the full research life-cycle from exploration, through proof and publication, to archival and sharing of data and code.

OpenDreamKit will be built out of a sustainable ecosystem of community-developed open software, databases, and ser- vices, including popular tools such as LINBOX, MPIR, SAGE (sagemath.org), GAP, PARI/GP, LMFDB, and SINGULAR. We will extend the JUPYTER Notebook environment to provide a flexible user interface. By improving and unifying existing build- ing blocks, OpenDreamKit will maximise both sustainability and impact, with beneficiaries extending to scientific computing, physics, chemistry, biology and more, and including researchers, teachers, and industrial practitioners.

We will define a novel component-based VRE architecture and adapt existing mathematical software, databases, and user interface components to work well within it on varied platforms. Interfaces to standard HPC and grid services will be built in. Our architecture will be informed by recent research into the sociology of mathematical collaboration, so as to properly support actual research practice. The ease of set up, adaptability and global impact will be demonstrated in a variety of demonstrator VREs.

We will ourselves study the social challenges associated with large-scale open source code development and publications based on executable documents, to ensure sustainability.

OpenDreamKit will be conducted by a Europe-wide steered by demand collaboration, including leading mathematicians, computational researchers, and software developers with a long track record of delivering innovative open source software solutions for their respective communities. All produced code and tools will be open source."

}

Chapter 3

Bibliography

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Chapter 4

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